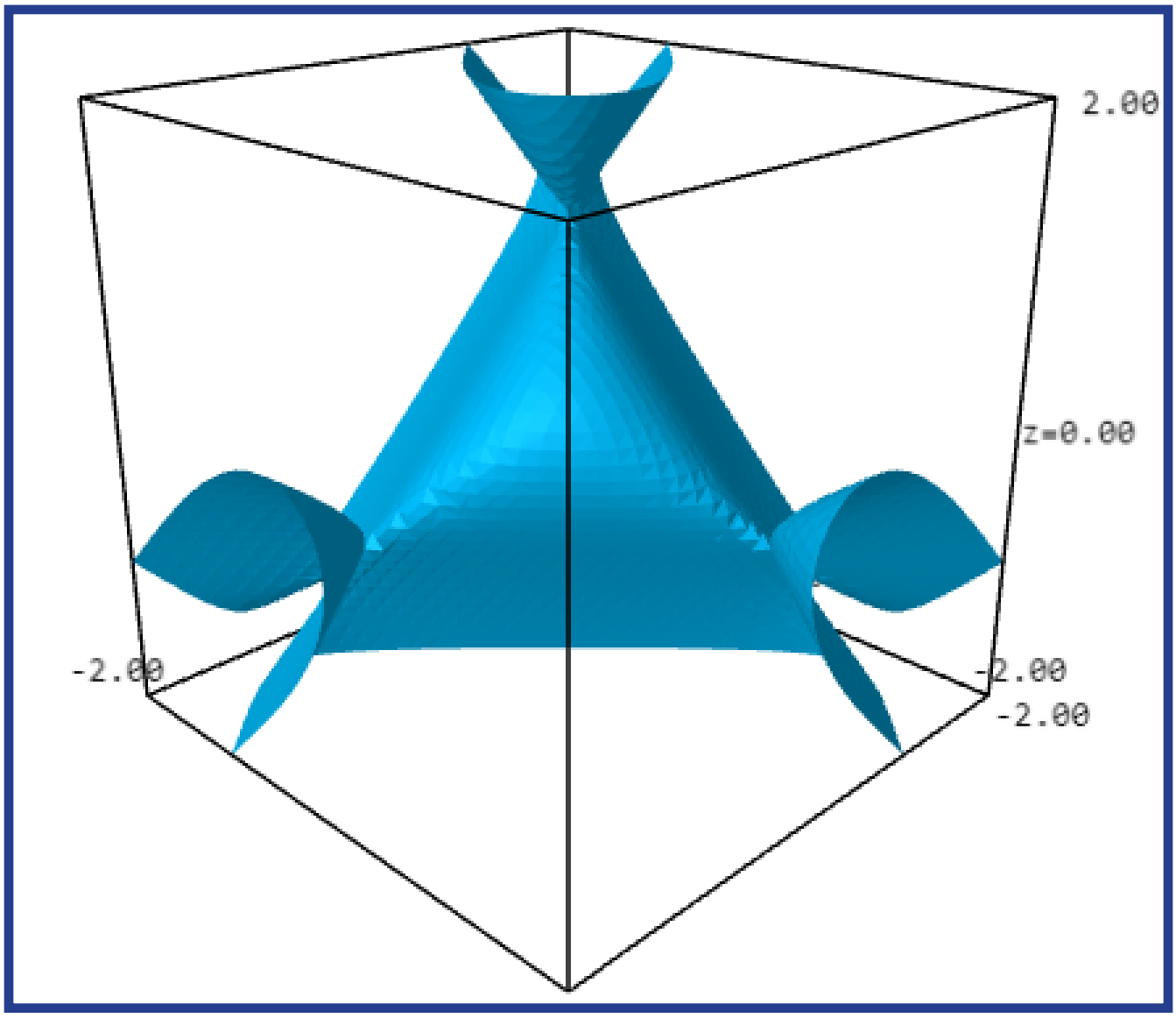
Autumn 2021

AIMatters

Newsletter of the American Institute of Mathematics



MATCHing Mathematicians AIM's New Virtual Classroom Program

Mixing Theory & Computation Software for Algebraic Geometry

Latinx Mathematicians Launching New Research Community

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Letter from the Director

ARCs, SQuaREs, Circles, and More

Greetings from San Jose! We learned how to run virtual programs this past year! We had 15 virtual workshops and hosted over 40 virtual SQuaREs, all in a virtual office space platform called Sococo. We also started a new program called AIM Research Communities (ARCs). An ARC is a group of 50–100 mathematicians in the same field who meet virtually in a large Sococo space for seminars and colloquia, problem sessions, reading groups, social hours, and collaborative research. This kind of effort at ongoing research in a virtual community is expected to continue long after COVID stops being a day-to-day concern. Perhaps it is a positive outgrowth of enduring this experience.

During these difficult times our outreach programs have also had online success. We've seen the development of the mathcommunities.org website and the large number of activities that are featured there, including Math Teachers' Circles, Math Circles for Students, the Global Math Project, Math Mondays, and a new activity: Virtual Classroom Visits. The last affords an opportunity for a mathematician to virtually visit a classroom in a socioeconomically disadvantaged school, with the chance to engage with students in a rich math activity. With the support of a Tensor SUMMA grant from the MAA, we are pilot-testing this program right now under the direction of Javier Haro with a group of 10 mathematicians who are experimenting with how best to conduct these visits.

I am happy to report that we've now started having in-person visitors: three brave SquaREs and a workshop, "Moduli spaces for algebraic dynamical systems," have met at AIM over the past two months. We managed to capture the spirit of AIM meetings in our virtual efforts to some extent, but the refrain we heard repeatedly as we got people back together was how happy they were to finally be doing math face-to-face (or perhaps I should say mask-to-mask) with their colleagues once again! We also all eat lunch outside now, which is quite pleasant.

We are pleased to announce that we have added five new Scientific Board members this year. They are Peter J. Bickel (University of California, Berkeley), Antonio Montalbán (University of California, Berkeley), Lenhard Ng (Duke University), Joseph Teran (University of California, Davis), and Melanie Matchett Wood (Harvard University). We are also happy to welcome Michelle Manes from the University of Hawaii to the AIM team. She will be working half-time as Deputy Director.

Finally, I am sorry to say that Fry's Electronics has closed its doors after more than 35 years of success. As you know, the AIM facility is housed in a corner of the (former) headquarters of Fry's Electronics in San Jose. We are still there (it's a little quieter there now, though our games area does liven things up a bit!) and will be for the coming months. We are actively engaged in securing a new location for future AIM meetings and activities. Stay tuned! ■

Brian Conrey

American Institute of Mathematics

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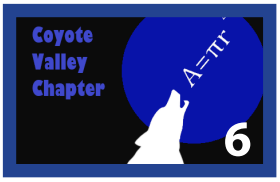
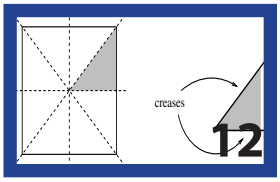
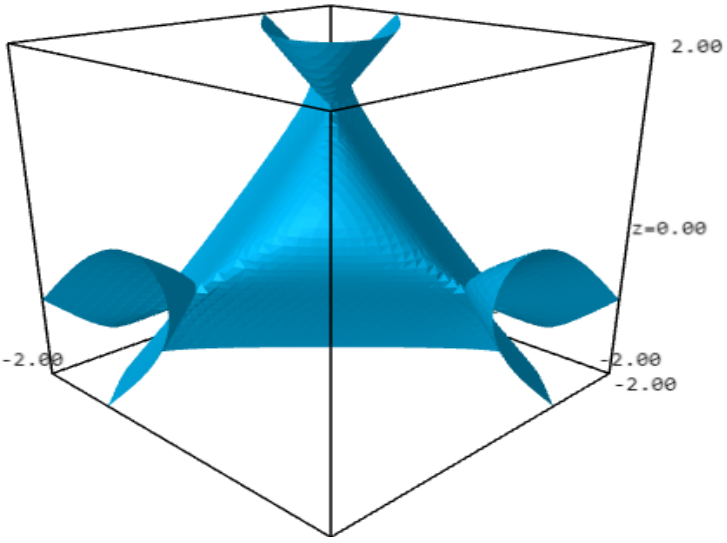






ABOUT THE COVER IMAGE

The cover image shows a unique cubic surface with three A2 singularities.
See p. 12.

The Math Circle Network

A National Hub for Student & Teacher Math Circles

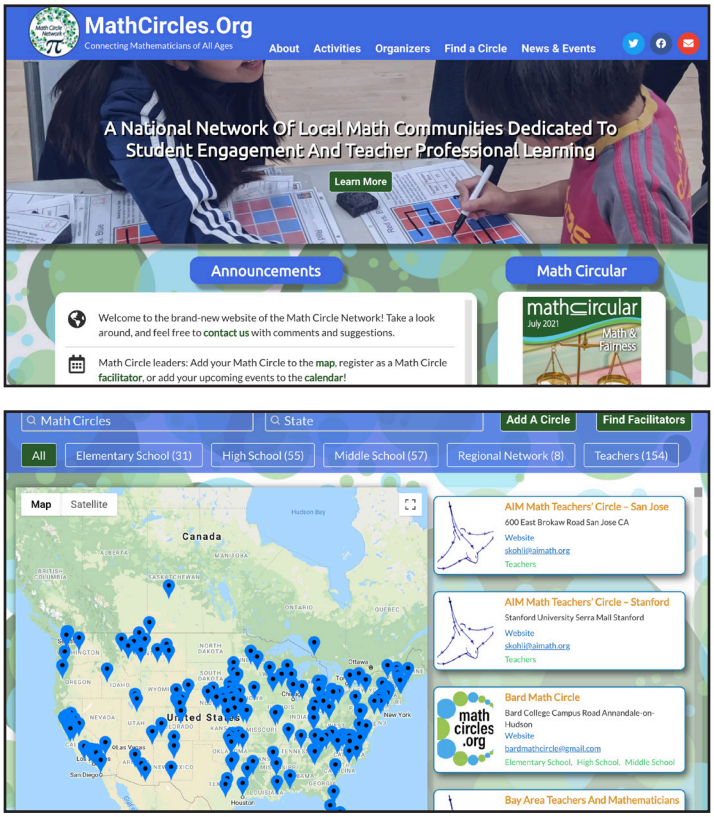
Math Circles are communities focused on the enjoyment of mathematical problem solving. Meetings are lively, interactive, and often "funstrating": challenging, but in a highly rewarding way!

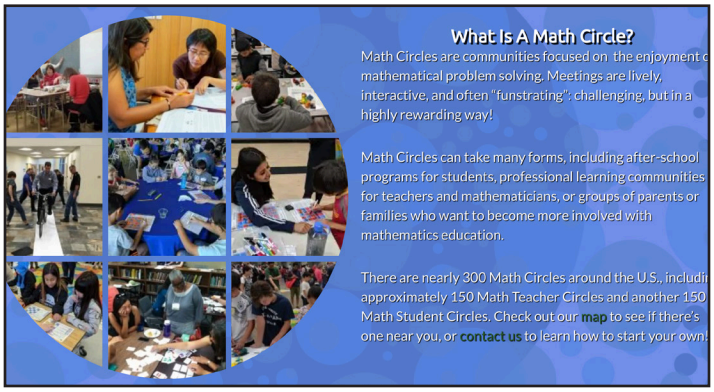
Math Circles can take many forms, including after-school programs for students, professional learning communities for teachers and mathematicians, or groups of parents or families who want to become more involved with mathematics education. There are nearly 300 Math Circles around the U.S., including approximately 150 Math Teacher Circles and another 150 Math Student Circles.

Having hosted the Math Teachers' Circle Network since 2006, in 2020 AIM became the host of the National Association of Math (Student) Circles, an organization founded by the NSF-supported Mathematical Sciences Research Institute. In order to better support the Math Circle community and foster connections among Math Circles for students and teachers, AIM founded the Math Circle Network in 2021.

We invite you to explore our brand-new website at MathCircles.org. Be sure to also check out our free publication, the *Math Circular* (www.MathCircles.org), a general-interest magazine for leaders and members of Math Circles and other math engagement programs.

– Brianna Donaldson





Images from the new Math Circles website, www.MathCircles.org.

Alexanderson Award

2020 Recipient Lecture & Ceremony

The third Alexanderson Lecture was presented by Laura DeMarco (Harvard Univesity) on September 30, 2021, at the Recital Hall at Santa Clara University. Due to COVID concerns, the lecture had been delayed a year. Laura also accepted the third Alexanderson Award on behalf of herself, Holly Krieger (University of Cambridge), and Hexi Ye (Zhejiang Universtity). The award recognized their seminal paper, "Uniform Manin-Mumford for a family of genus 2 curves," which was published in the Annals of Mathematics. The paper grew out of the AIM SQuaRE, Dynamical Andre-Oort Questions.

The award and lecture is given in honor of Gerald ("Jerry") Alexanderson, Professor of Mathematics at Santa Clara University and founding chair of AIM's Board of Trustees. The ceremony was bittersweet since Jerry died on December 16, 2020. His influence was instrumental in AIM's history and development from the beginning more than 25 years ago and continuing to this day. ■

– Estelle Basor

Gerald Alexanderson, 1933-2020

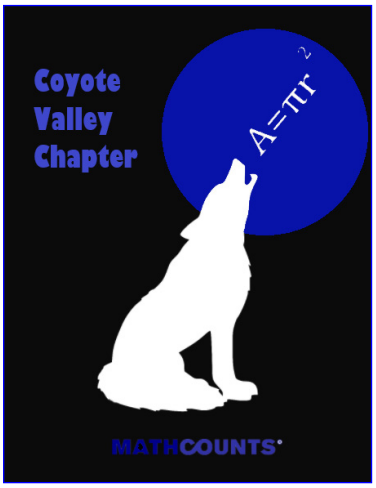
Gerald Alexanderson was a member of the Santa Clara University faculty beginning in 1958 until his retirement in 2018. During that time, he was Chair of the Mathematics department for 35 years and a member of the Faculty Senate Council. For 38 years he held the endowed Valeriote Professorship of Science Chair. He was the author of more than a dozen books, including textbooks in abstract algebra, and discrete and combinatorial mathematics. Alexanderson was the first recipient of Santa Clara University's Bayma Award for Scholarship, and he received the Special Appreciation Award from the Dean of Arts and Sciences as well as the Special Recognition Award for Teaching, Research, and Service from the President of the University.

Alexanderson's influence extended to the national level, where he played a leading and lasting role in the Mathematical Association of America (MAA). His contributions to the MAA spanned more than 50 committees and 24 years on the Board of Governors, encompassing Secretary, Vice-President, and President of the Association and Editor of *Mathematics Magazine*. Results of this work included the remodeling of the MAA Carriage House in Washington, D.C., into its Mathematical Sciences Conference Center. In this time, Jerry served on the Science Policy Committee of the American Mathematical Society (AMS) and was a consultant to the Editorial Board for the Bulletin of the AMS. In testament to his expansive record, Alexanderson received the MAA's most prestigious award for distinguished service to Mathematics, the Yueh-Gin Gung and Dr. Charles Y. Hu Award.

Dispatches from Morgan Hill
Math Enrichment Flourishes Online

The 2020-21 school year began online for all local students, so the Morgan Hill Math programs did as well. Morgan Hill Math is an outreach program sponsored by AIM, which provides free math enrichment activities to about 300 students each year, who live in or near Morgan Hill, California.

It is now difficult to remember the uncertainty of August, 2020. *Will numbers go down? When will students return to class? How do we teach over Zoom?* Sadly, the decision was made to skip the Mathletics and MathCounts6 programs for 2020-21. However, MATHCOUNTS, Math Olympiad, the AMC contests, and Math Madness were offered over Zoom to all interested students.



Coyote Valle MATHCOUNTS chapter's logo.

The cornerstone of the Morgan Hill Math program has always been training for and competing in MATHCOUNTS. This past year I offered virtual MATHCOUNTS training for the season. Over 70 students, from nine local schools, trained with me once a week, on various days, from September through December, sharpening their skills in counting, statistics, proportions, probability, permutations and combinations, functions, algebra, and geometry. The MATHCOUNTS Foundation, in cooperation with The Art of Problem Solving (AoPS), offered monthly, online practice exams to help with training, and to prepare the students to take competitions online. Forty-two students participated in the MATHCOUNTS Coyote Valley Chapter Competition, 12 students competed in the new Chapter Invitational Competition, and five students qualified for the California State Competition!

All interested students from the previous year's programs, currently in third through eighth grades, were invited to participate in one or both divisions of the Math Olympiad for Elementary and Middle School (MOEMS). From November to March, over 80 kids were challenged to strengthen their problem solving skills in this once-a-month, five-question test. MOEMS transitioned the contest to be given online. This online option continues to be available, and we will take advantage of it. Online enables more students to participate!

I wanted another activity for the elementary students, so we competed in the weekly Math Madness competition in the fall of 2020. As a joint initiative between American Mathematics Competitions (AMC) and AreteLabs, Math Madness is emerging as one of the premiere math competition events in the United States. About 20 kids in third through fifth grades competed October through December, and they finished in ninth place overall! We competed against teams from all over the country, every week. After each match, I held a Zoom meeting to go over the questions and solutions. After the MATHCOUNTS students concluded their season in February, many of them still wanted to compete, so we formed a Math Madness Middle School team and did the Spring competition, coming in fifth place overall! When the official tournament was over, the students wanted to continue. So we held weekly competitions through the summer. These kids thrive on every challenge I give them!

Our Middle School and High School Mathletes participated in several other competitions this year. All students involved in the weekly MATHCOUNTS training classes, as well as interested fifth and sixth graders, took the AMC8, an MAA competition for students in eighth grade and below. The MAA in cooperation with AoPS, developed a platform which allowed students to take the contest online. Seventh-grader Aryan Ganesh scored in the top 5%, nationwide, earning him a spot on the Honor Roll. A few of these middle schoolers also challenged themselves and

joined high school students in taking the AMC10 or AMC12 exams. By scoring high enough, three local students, Neil Shah (11th grade), Antarish Rautela (11th grade), and Aryan Ganesh (seventh grade) qualified to take the AIME, the American Invitational Mathematics Exam, which is the first in a series of examinations that culminate with the International Mathematical Olympiad (IMO). Neil received the Certificate of Distinction for scoring in the top 5%, nationwide, on the AMC12. Additionally, two teams of six students competed in the online Purple Comet! Math Meet.

In April 2021, the Morgan Hill Student Math Circle for Elementary and Middle School students was formed. We met every Monday over Zoom to explore fun math activities. Most activities were found on the Math Communities and JRMF webpages. The Morgan Hill Math Teachers' Circle was on hiatus for the school year. I look forward to its return.

In May 2021, the Morgan Hill Unified School District identified advanced math students in third through fifth grades, and they were invited to try out for a spot in the fall 2021 Morgan Hill Math programs, Mathletics and MathCounts6. Tryouts were held over Zoom, and we are all looking forward to resuming our after school math enrichment activities. The families and the coaches are excited to meet in classes again. It is so rewarding to generate that spark and encourage the next generation of students to fall in love with math. ■

– Kelley Barnes

MATCHing Mathematicians

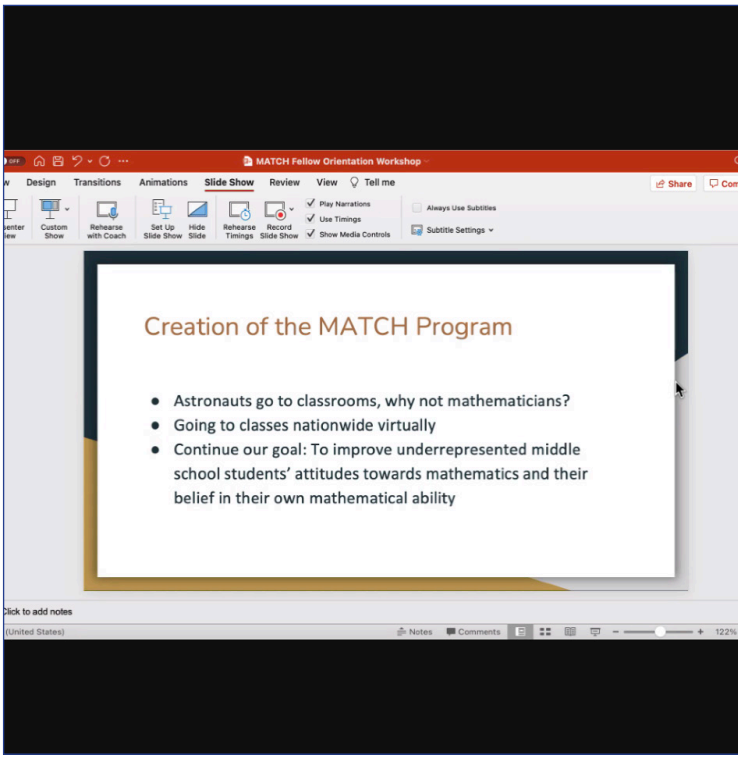
AIM's New Virtual Classroom Program

AIM is excited to announce a new program, "MATCH," that pairs mathematicians interested in K-12 outreach with middle school classrooms serving (primarily) students historically underrepresented in mathematics via recurring "virtual classroom visits." After receiving training in working with young students, participating mathematicians present interactive online lessons based on well-tested and engaging Math Circle style activities. They also have the opportunity to talk about their experience as a mathematician. By increasing access to fun and creative math activities and positive mathematics role models, the program aims to improve students' attitudes toward mathematics, belief in their own mathematical ability, and their overall likelihood of pursuing a career in mathematics.

MATCH grew out of AIM's K-12 initiative, www.MathCommunities.org. Started in March 2020 in response to the COVID pandemic, MathCommunities.org provides a variety of free resources to schools and educators. During the 2020-2021 school, the Math Communities team, led by MATCH Program Director Javier Haro, conducted several dozen "virtual classroom visits" at Title I schools, with overwhelmingly positive feedback. For example, one seventh grade teacher wrote:

> ***"Thanks so much for taking the time to come and virtually meet my students. They enjoyed your lesson on Gerrymandering and learned the real life math applications. This was the first time in 17 weeks that all students had their cameras on, no one had wifi issues or tech issues (it all happened magically). My students found your lesson very engaging and some of them got a peek into applied mathematics. I so appreciate your commitment to making math enjoyable for all."***

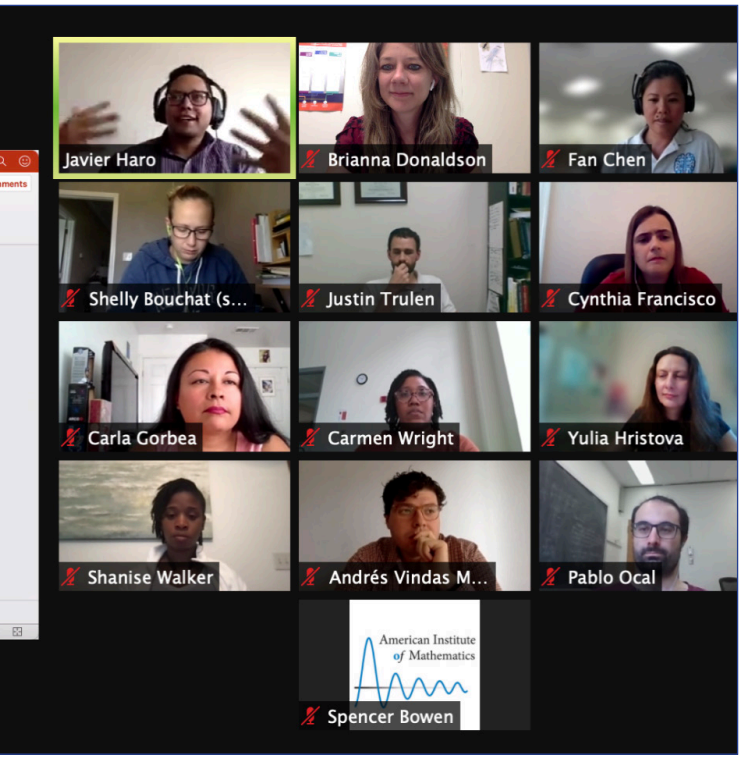
Due to the positive response to last year's virtual classroom program, AIM applied for and received an MAA Tensor SUMMA grant to expand the program during 2021-2022. Now known as "MATCH," the program is pairing 10 mathematicians, or "MATCH Fellows," with middle school classrooms at socio-economically disadvantaged schools. Interest from both mathematicians and teachers has been extremely strong, with the program receiving nearly five times as many applications as could be accommodated this year!



Each MATCH Fellow will visit their classroom three times during the 2021-2022 school year. "I'm excited to have our MATCH Fellow mathematicians in the classrooms because I witnessed their enthusiasm to teach from this summer's training workshop," said Program Director Haro. "The way that we have structured each classroom visit will benefit students in showing them mathematics (applied, theoretical, and societal) that is not typical in a traditional mathematics

classroom as well as learning more about what mathematicians do."

Although MATCH is timely due to the current remote learning situation, it has long-term potential benefit. According to AIM Executive Director Brian Conrey, "We believe that the model has strong potential as a low-cost but high-impact mechanism for mathematicians to get directly involved in K-12 education." For example, the Alliance of Indigenous Math Circles sends mathematicians to Indigenous communities to work with students. They have expressed



interest in using virtual visits to expand their outreach potential. Other networks of minority mathematicians have also discussed the potential of virtual classroom visits to help connect mathematicians of color with students of color. Finally, education leaders working with students in highly rural areas have also expressed interest in virtual visits as an exciting opportunity that their students simply would not have access to in person.

NASA has a program for schools to receive remote visits from astronauts. AIM believes that with engaging materials, students all across the country can be just as excited by a virtual visit from a mathematician!

Congratulations to the inaugural class of MATCH Fellows!

- Rachelle Bouchat, Berea College
- Fan Chen, El Paso Community College
- Cynthia Francisco, Oklahoma State University
- Carla Gorbea, California State University Dominguez Hills
- Yulia Hristova, University of Michigan-Dearborn
- Pablo Ocal, University of California, Los Angeles
- Justin Trulen, Kentucky Wesleyan College
- Andrés Vindas Meléndez, University of California, Berkeley, and MSRI
- Shanise Walker, University of Wisconsin-Eau Claire
- Carmen Wright, Jackson State University

– Brianna Donaldson

mathcommunities.org

From Undergrad Student to PhD

Doug Knowles and REUF

Research Experience for Undergraduate Faculty, or REUF, encourages and supports involvement in research with undergraduates by faculty at colleges and universities that emphasize undergraduate education. Funded by the National Science Foundation, the program is a collaboration between AIM and another of the NSF math research institutes, ICERM, located at Brown University in Providence, RI.

Patrick Rault

Each annual cycle of REUF includes a week-long workshop during the summer, along with competitively funded activities for participants afterwards to support continuation of research engagement sparked by the workshop. Some REUF groups grow into long-term collaborations. One such group, which started after a REUF workshop in 2011, recently grew to include a brand-new math PhD who began working on this REUF project as an undergraduate: Doug Knowles.

In the 2014-15 academic year, when Knowles was an undergraduate at SUNY Geneseo, his professor Patrick Rault, a member of the 2011 REUF continuation group, received a Center for Undergraduate Research in Mathematics (CURM) grant to work with a group of undergraduates on a project about numerical ranges, which was also the basis of the continuation group's project. Four undergraduates, including Knowles, then helped the continuation group (which included Kristin Camenga, Patrick Rault, Ilya Spitkovsky, and Rebekah Johnson Yates) with their research. Inspired in part by the numerical ranges project, they have now all earned their PhDs: three in mathematics and one in physics.

As of Summer 2021, Knowles is now formally a part of the ongoing REUF continuation group. When asked what he thought of his own growth as a mathematician through REUF, Knowles commented:

> ***"My time with REUF really helped my confidence; the group always took the time to listen and think to my contributions. I lost my fear of speaking up in a group to present ideas. I think when it's only two people working on a project you don't get the same level of group-wide collaboration and conversation that you do when there's more than two people involved. Getting to have that experience with REUF really helped my confidence as a researcher and developed those collaborative skills that are really crucial to a mathematician."***

Overall, REUF alumni have mentored several hundred undergraduate students in research projects. In Knowles's case, the program had a crucial influence on his present career and will continue to have an impact on his professional life as the group continues their research. ■

– Sonya Kohli

Douglas Knowles, 2015 CURM conference

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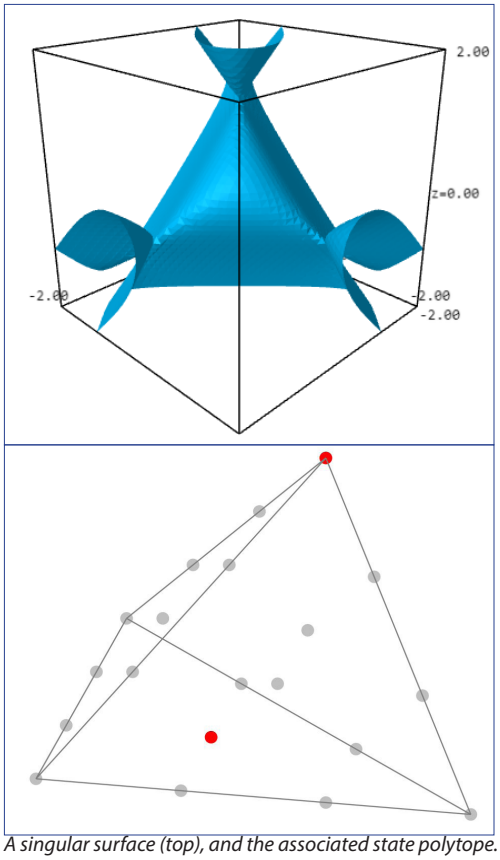
Please find us at JMM 2022 in Seattle.
Visit the AIM booth in the exhibits hall
and join us for the
Mathematical Institutes Open House on
Thursday, January 6th from 6:00-8:30 PM.

Mixing Theory and Computation

New Software for Algebraic Geometry

The AIM SQuaRE "Computational aspects of GIT with a view of moduli spaces" recently had its third week-long meeting, a virtual meeting due to COVID. The team, consisting of Patricio Gallardo (University of California, Riverside), Jesus Martinez-Garcia (University of Essex, UK), Han-Bom Moon (Fordham University), and David Swinarski (Fordham University), produced a new tool for research in algebraic geometry.

An ancient goal of mathematics is to classify objects based on their geometric properties. The modern approach is to use techniques from another area of mathematics, often algebra, to solve what appears to be inherently a geometry problem. Recent progress on manipulating systems of equations has vastly simplified that problem in many cases. The research concerns Geometric Invariant Theory (GIT), which uses a system of equations to describe a landscape of geometric objects. That is, each solution to the equations describes one object, and every object corresponds to one solution. This enables one to use the tools of algebra to study a problem in geometry. Now, having turned a geometry problem into an algebra problem, is the transformed problem easier? Thanks to new research, it is—in some cases.



A singular surface (top), and the associated state polytope.

The starting point of GIT is a system of equations, but that system will be redundant: it will have extra symmetry which does not accurately reflect the underlying problem. Removing that redundancy introduces a new problem: some of the equations have undesirable properties which make the system difficult to analyze. Those bad equations describe the "unstable locus." Identifying and removing the unstable locus is the task which was automated by the SQuaRE group. The work of the SQuaRE involved both devising a theoretical algorithm, which can be used in a wide range of settings, and implementing that algorithm in computer code.

The implementation will be available, for free, for anyone to use. More than a dozen recent research papers have used an ad hoc approach to deal with special cases of this algorithm, often involving considerable effort. The new computer code is able to do all of those previous calculations in a matter of minutes. More significantly, the software and its underlying algorithm have been thoroughly tested, giving a high level of confidence in the result. In addition to verifying previous work, the new code handles several new cases which were not accessible by previous methods.

The new software will facilitate the reproducibility of certain mathematical proofs—the ability for a third party to check the results. Reproducibility is fundamental to scientific research, including mathematics. For traditional proofs the written words provide a complete argument, enabling a knowledgeable reader to verify that the proof is correct. But if a portion of the proof involves a computer calculation, verification is difficult—particularly so when the calculation involves specialized code. The current work produced general code which applies in a multitude of cases. This actually makes it easier to check that the code is correct, and thus makes it possible to verify the results in any specific case.

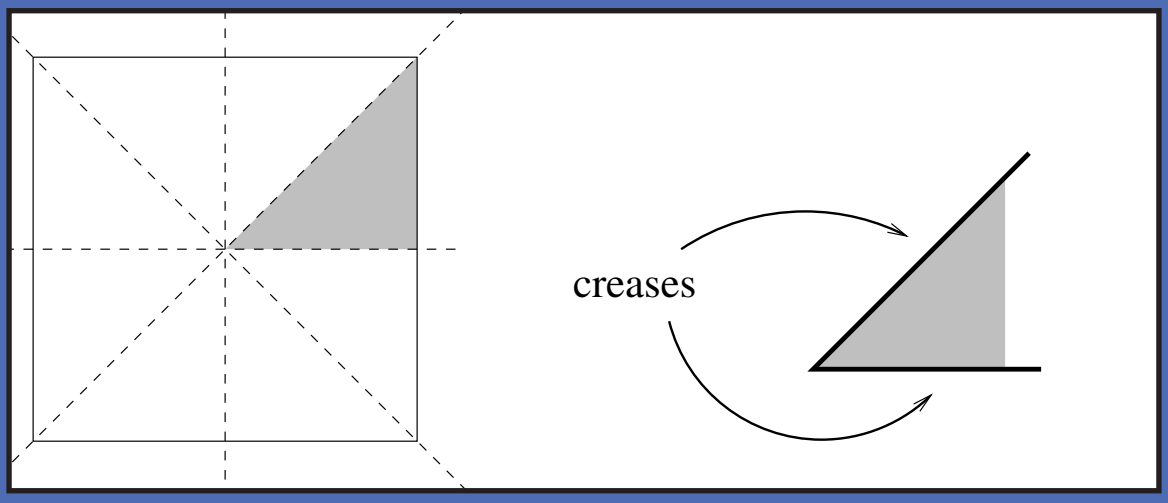
The algorithms have been implemented in SageMath, a free and open source computer algebra system, which serves as a fundamental tool for research in many areas of mathematics. A paper describing the work is in the final stages of preparation. ■

– David Farmer and Sally Koutsoliotas

The problem solved by the SQuaRE group can be understood by analogy with folding a piece of fabric or wrapping a package with paper much larger than the box it will cover. No matter how hard you try, you will end up with wrinkles or creases.

A square of paper illustrates the problem. A square has eight symmetries: four rotation symmetries, and four reflection symmetries. Those symmetries correspond to the eight ways the shaded triangle in the figure can be moved to another location in the SQuaRE.

The symmetries can be interpreted as instructions for folding up the square. Once all the folds are made, the square has been transformed into a triangle, which is 1/8th the size of the original square. Along two edges of the triangle are creases, and (from a mathematical perspective) creases are difficult to analyze because they have a different structure than most points in the triangle.



The same situation arises in GIT: the original set of equations has symmetries, and once one accounts for those symmetries (the analogue of "folding up" a square), there are "unstable points" (the analogue of creases). The new algorithm automates the process of describing the folded up equations in a way that avoids the creases.

Latinx Mathematicians
Launching a New Research Community

In June 2021, sixty mathematicians gathered in a virtual AIM space for an intensive week-long workshop that launched the Latinx Mathematicians Research Community (LMRC; https://aimath.org/programs/researchcommunities/lmrc/). The LMRC grew out of a series of two "Latinx Mathematicians Network" planning workshops that were organized by Pamela E. Harris (Williams College) and Jesús De A/Loera (UC Davis), and funded by a supplemental grant to AIM from INCLUDES, an NSF-wide "Big Ideas" program aimed at increasing participation by underrepresented groups in STEM fields.

According to Harris, the organizers' vision for the first workshop, held in December 2018, was to bring together the community to identify challenges faced by Latinx and Hispanic mathematicians in succeeding at the highest levels in research and leadership. The second workshop, held in December 2019, brought in an additional focus on education. Taken together, the workshops pointed toward the need to mentor early-career Latinx mathematicians by providing research and professional development opportunities.

"A lot of the younger folks were being asked to do a lot of service and outreach activities, rather than being invited to contribute and to collaborate with other people in research. After the second workshop, we thought the best thing to do would be to have a year-long program, similar to AIM SQuaRE research groups, where we could bring together Latinx mathematicians and give them the space and time to think about mathematics deeply, build community, talk about how to secure grants and how to apply for jobs," said Harris.

The LMRC currently includes 12 senior Latinx research mentors and 48 early-career Latinx researchers working on five different topics:

- Measuring Polytopes, led by Federico Ardila, Pamela E. Harris, and Guido Montufar
- Codes from Geometry, led by Adriana Salerno and Tony Várilly-Alvarado
- Teaching and Learning of Mathematics—The Role of Body Movement, led by Vilma Mesa and Hortensia Soto
- Finding Patterns in Unexpected Places, led by Stephan Ramon Garcia and Victor H. Moll
- Data-Driven Models for Pandemic Related Decision Making, led by Luis David Garcia Puente, Jesús A. De Loera, and Sara Del Valle

The LMRC also includes a service component, a monthly webinar series called "Hidden NORMS" (www.hiddennorms.com), co-organized by Harris, Kimberly Hadaway, Daniel Qin, Vanessa Rivera Quiñones, and Dwight Williams II. Hidden NORMS introduces undergraduates, especially those from underrepresented groups, to what Harris calls "raw, real advice" about navigating the professional world of the mathematical sciences. The LMRC also has strong connections with the 501(c)(3) organization Lathisms (www.lathisms.org), co-founded by Harris, which showcases the contributions of Latinx and Hispanic mathematicians.

The research groups of the LMRC will continue to meet regularly throughout this year, and hopefully beyond. The whole community also gets together each month for professional development seminars as well as colloquia presented by the groups about their research progress. "Mathematics is a way to be a human. It is built into us, as is being a species that collaborates," said Harris. "We hope the LMRC continues for years." ■

– Brianna Donaldson

Another series of AIM workshops funded by the same grant, "Network of Mathematicians of Color," was organized by Shelby Wilson (now at Johns Hopkins University) and Michael Young (now at Carnegie Mellon University) with the goal of bringing together leaders of the Black mathematics community. Working together, the Black and Latinx workshop participants collaboratively developed a database of minority mathematicians to increase the visibility of and identify members of the minority mathematical community. The hope for the database is that it will promote connections, foster diverse research communities, and increase representation at conferences and workshops as well as for leadership positions and awards. Individuals who identify as members of the community of underrepresented minorities in the mathematical sciences are encouraged to register for a free profile at https://minoritymath.org/database/.

THANK YOU

AIM gratefully acknowledges the following donors for their generous contributions of books, reprints, journals, and archives to the AIM library in 2020 and 2021:

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Marc Holtz
Mary Jennings
Barbara Martin
Hugh Montgomery

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From Our Collections

The Fields Medal, 1936-1966
Key papers selected from the reprint collections of Tibor Rado, Paul Bateman, Andrew Gleason, Armand Borel, Herbert Federer, and Abraham Robinson