the view from aim 2024

Syzygies and Mirror Symmetry

A Hot Topics Workshop

Positive Math Identities

Supporting Diverse Students and Educators

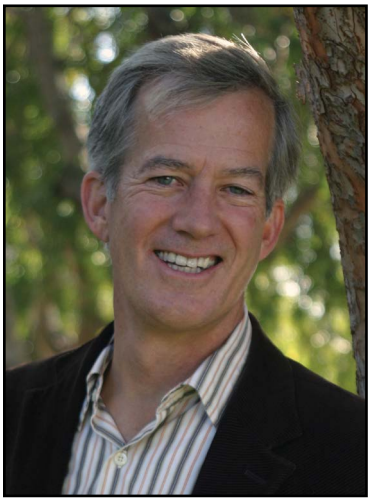
Joining Forces

AIM and Caltech Forge a New Collaboration

American Institute of Mathematics

Letter from the Director

Looking Back, Looking Forward



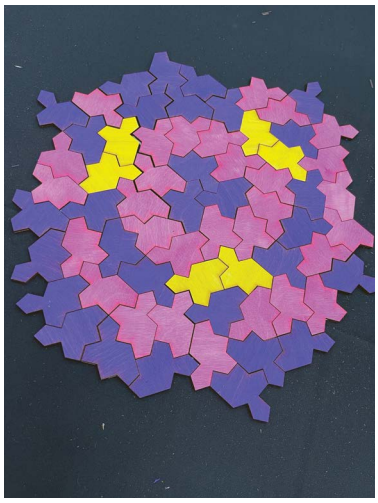
Greetings from Pasadena! This year has been very eventful, mostly because of our move to our lovely new facility on the 8th floor of Caltech Hall.

After Fry's Electronics closed up shop in 2021, we found ourselves all alone in a corner of the 200,000 square foot former corporate offices of Fry's. To keep the property from being taken over by uninvited participants, it was necessary to fence it off. To make it more welcoming for workshop and SQuaRE participants, we got some indoor and outdoor games, including ping pong and pickleball. Many people really enjoyed that relaxing aspect of AIM's last days in San Jose!

But now we have a brand-new home, remodeled thanks to the gracious contribution of Richard N. Merkin. It has a stunning view of the San Gabriel Mountains, atop which sits the Mount Wilson Observatory, where Edwin Hubble first discovered that the universe is expanding. It is inspiring to say the least. We expect a record number of proposals this year, as people are excited to see our new space!

Our first week of operations at Caltech was the week of July 10, when we had five SQuaREs, and we have had an event or two every week since! Thank goodness we have such a hardworking and dedicated staff!

One of the most exciting workshops we have hosted since the move was the INCLUDES workshop in September. It tested the limits of our space, as 55 participants spent three days thinking about how better to support positive math identities for all students and educators, especially those from underrepresented communities.



We haven't yet moved most of our games and puzzles, but we do have a set of 300 beautiful laser-cut wooden Spectre tiles (shown above at right), donated to us by one of our Math Communities partners, MathHappens. If you haven't heard of Spectre tiles, they were discovered in 2023 and can tile the plane but only in an aperiodic fashion. They are super fun to play with! If you want your own set, MathHappens has the instructions for how to make them on their website.

Although library donations were curtailed a bit as we moved spaces, I do want to mention that we received 4,000 reprints from Hugh Montgomery, many of them passed down from Hugh's advisor Harold Davenport, which contain some of the very best papers in analytic number theory of the last 100 years. Thank you, Hugh! Also, we have started collecting again, so please send us your valuable math artifacts!

We hope you enjoy this issue of our newsletter and look forward to seeing you in Pasadena soon! ■

Brian Conrey

American Institute of Mathematics

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the view from aim
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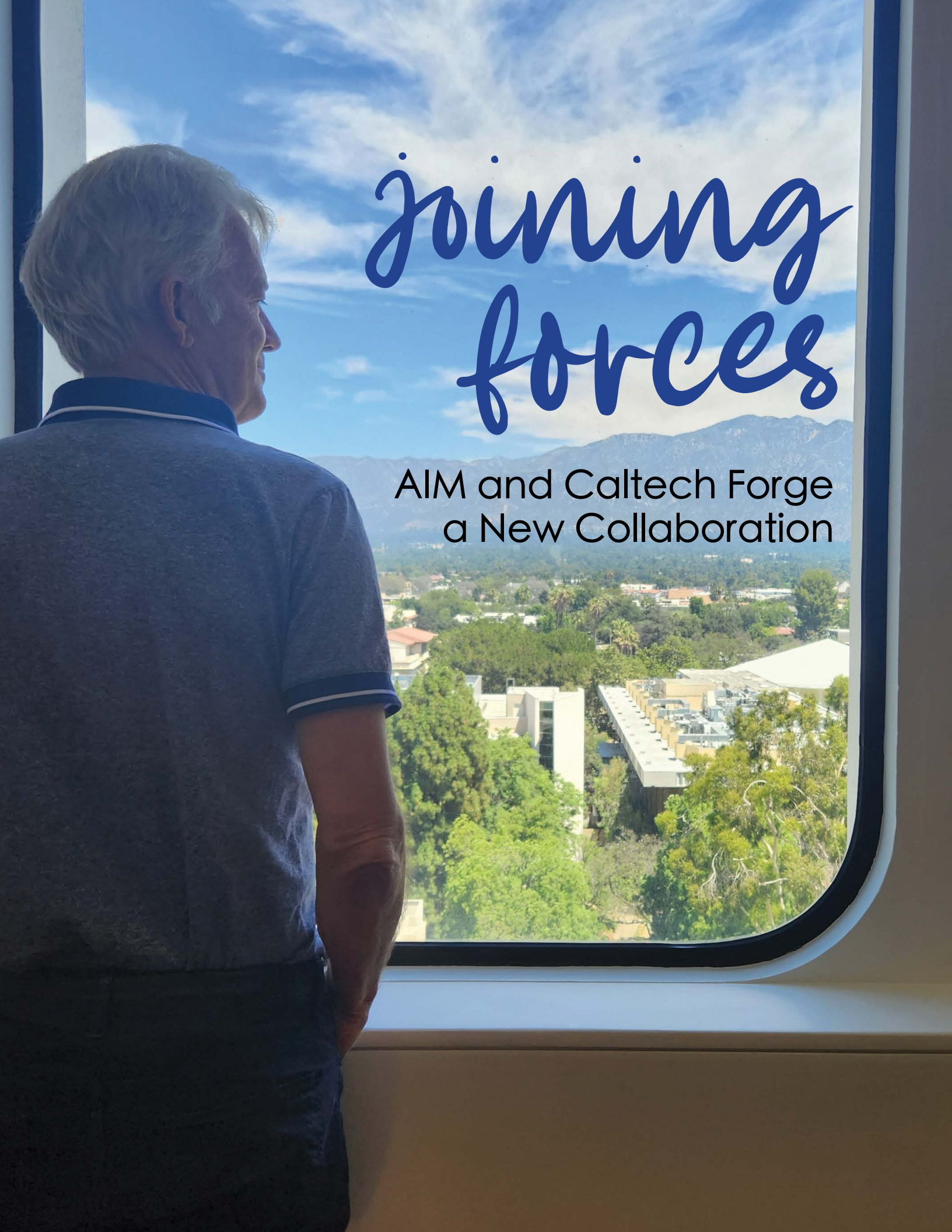
about the cover

The cover shows a portion of the wallpaper in the new Richard N. Merkin Center for Pure and Applied Mathematics. The dots around each circle represent the eigenvalues of random unitary matrices of various sizes (40 by 40 and 50 by 50). Eigenvalues of random matrices have shed light on many phenomena, from analyzing noisy data to the energy levels of large atomic nuclei. The eigenvalues in the wallpaper images have been used as a random model for the zeros of the Riemann zeta-function. The matrices used for the wallpaper were produced by David Farmer, Director of Programs at AIM, by finding the QR decomposition of matrices with random complex Gaussian entries. Photo by Terry Busk, Operations Manager at AIM.



joining forces

AIM and Caltech Forge a New Collaboration

from Brian Conrey

Executive Director of AIM

When Fry's announced in February 2021 that they were closing their doors, we embarked upon finding a new home for AIM. The NSF encouraged us to partner with a university, so we floated the idea to several universities in northern California. This was well into the pandemic, so we hadn't had any on-site programs in quite a while. In fact, between March 2020 and January 2022, there was only one week when any workshop or SQuaRE participants came to AIM. That was the week of August 17, 2021. In one of the three brave SQuaREs that came that week was Omer Tamuz from Caltech.

Omer asked me what was happening with AIM, and I told him about my discussion with the nearby universities. He asked me if he could investigate whether there was any interest from Caltech in having AIM there. A little while later, we were informed that there was interest, and Estelle Basor (our then-Deputy Director) and I were invited down for discussions. We met with the President, the Provost, and several division directors and department chairs, all of whom expressed how much they wanted AIM to come to the Caltech campus. Fiona Harrison, the Division Director of Physics, Math and Astronomy (PMA), told me that she had spoken with 40 individuals (faculty and graduate students) from 5 of the 6 divisions at Caltech who had been to programs at AIM, and that every one of them said words to the effect of, "If you can get AIM to Caltech, do it!"

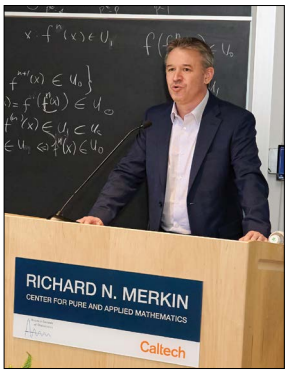
Lounge area in the Merkin Center, AIM's new home.

One of AIM's interests was to remain an independent organization, which Caltech was able to accommodate. In addition, they said they had a donor who would remodel space for us for (almost) immediate use (thank you, Richard N. Merkin!), and that over the next few years they would remodel a second space for us in Kellogg Hall, adjacent to the Math Department in Linde Hall. It was truly an overwhelming, beautiful offer, one that we could not pass up!

In January 2022, we formally accepted Caltech's offer. I joined a committee tasked with redesigning the space on the 8th floor of Caltech Hall. The space had been gutted—no flooring, no walls, no ceilings—a tabula rasa, so to speak. The committee met diligently and came up with what I think is a spectacular new home for AIM, with exceptional views out the window, right in the heart of an exquisite campus and next to a glorious turtle pond. What a joy!

from Sergei Gukov

Director of the Richard N. Merkin Center for Pure and Applied Mathematics and John D. MacArthur Professor of Theoretical Physics and Mathematics at Caltech



On behalf of the Merkin Center, I am excited to welcome AIM to the Caltech campus! It has been a very eventful journey and one that is particularly meaningful to me, having served on the AIM Scientific Board for many years and participated in all types of its programs.

Labor Day 2023 marked exactly two years since discussions began about bringing AIM to Caltech. That weekend, we unloaded the last of the AIM furniture from the moving truck. As I carried boxes to the

8th floor of Caltech Hall, I couldn't help but feel an overwhelming sense of excitement and disbelief that we had actually succeeded in moving a national math institute to our campus!

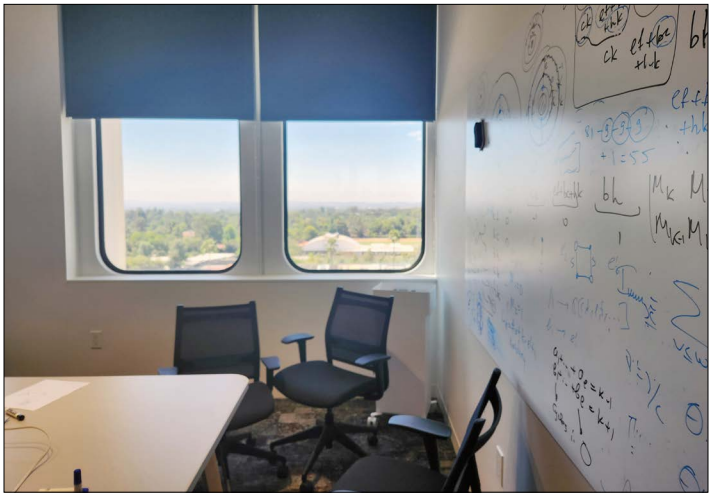
At the start of this journey, we never could have imagined bringing AIM here to Caltech. The stars aligned and the work of so many people helped bring this ambitious goal to fruition. Timing was in our favor, as AIM happened to be searching for a new home as Caltech was making plans to further invest in mathematics. Our visionary leaders —PMA Division Chair Fiona A. Harrison, Provost David A. Tirrell, and President Thomas F. Rosenbaum — swiftly responded to this opportunity, as it had the potential to advance Caltech's prominence in mathematics.

We had to determine how we would make this transition happen. We were faced with many questions and problems to be addressed. We had to determine where we would house AIM and where we would find the funds to support it. We also had to answer whether we wanted AIM to become a part of Caltech or remain as a separate organization and, if it were to remain separate, how we would handle its finances.

From the start, there has been unanimous and enthusiastic support for AIM across all levels and disciplines, extending far beyond pure mathematics. "By bringing AIM to Caltech, we can increase the rate of serendipitous encounters of mathematicians and quantum scientists and accelerate the progress at this exciting math-quantum interface," said Hirosi Ooguri, the Fred Kavli Professor and the Director of the Walter Burke Institute for Theoretical Physics. Such levels of support across disciplines turned out to be instrumental not only at the initial stage but also further down the road, including in July 2023, when AIM needed a temporary space.

Wikimedia Commons

Caltech Hall, the tallest building on campus at nine stories. AIM's home is on the 8th floor.



A breakout room with a view.

My own experience serving on the AIM Scientific Board and familiarity with AIM's programs, structure and impact allowed me to provide input at the early planning stages and to better organize the work of the Merkin Center for Pure and Applied Mathematics to assist with the move. The Center, endowed through a generous gift of Caltech Trustee Richard N. Merkin, now has the capacity to host AIM and to run other activities.

The leadership of other Mathematics Institutes (including the Director of MSRI at the time, David Eisenbud, the Founding Director of ICERM Jill Pipher, and the former Deputy Director Elisenda Grigsby) also provided valuable input that helped us find the right business model for the partnership between AIM and Caltech, anticipating many logistical nuances of AIM's operation on the Caltech campus.

With its operational model in place, the physical relocation of AIM was the final step in completing the transition. The move itself went smoothly, with no major disruptions to AIM activities. This couldn't have happened without the dedication of the AIM staff and Caltech support staff who helped set up the facility, technology and a myriad of other tasks. Thank you to Carol Silberstein, Michelle Vine, Shawna Silesky, Esperanza Madrigal, Marcella Ta and Denise Lu, among others, for helping to make this move happen!

With the transition of AIM complete, we recently hosted an open house introducing AIM to the Caltech community. The event was a success and helped forge new connections. We are off to a great start and am excited for what's to come! It's been an honor to be a part of the incredible AIM team and I look forward to many years of our continued collaboration. ■

AIM Welcomes PRIMES Fellows

Two Faculty Members Win Prestigious NSF Awards

Mary Flagg and Veronika Furst are changing their students' lives through mathematical research. Many of the students that they teach are the first in their families to attend college and have not previously been exposed to such research. After their involvement in undergraduate research projects, some go on to graduate school and careers as mathematical researchers and teachers. Furst and Flagg are both AIM PRIMES Fellows who will spend several weeks at a time in residence at AIM over the next two years. They recently discussed their goals with their PRIMES co-PIs, Brianna Donaldson and Leslie Hogben.

The new NSF program Partnerships for Research Innovation in the Mathematical Sciences (PRIMES) is intended to build research capacity at Minority Serving Institutions, including University of St. Thomas in Houston, Texas, and Fort Lewis College in Durango, Colorado, where Flagg and Furst are faculty members. The program emphasizes faculty members' research, impact on their students, and national impact.

From left to right: Hogben, Furst, Flagg, and Donaldson in front of Caltech Hall, AIM's new home

Donaldson, Flagg, Furst, and Hogben all met through the AIM and ICERM Research Experiences for Undergraduate Faculty (REUF) workshop in 2015, organized by Donaldson, Hogben and Ulrica Wilson of ICERM. Flagg and Furst were members of a research group led by Hogben that worked on zero forcing and power domination, a collaboration that initially produced two papers. Flagg and Furst both redirected their research programs into the area of the inverse eigenvalue problem of a graph and zero forcing (IEPG-ZF), where problems are accessible to undergraduate researchers. Each has since supervised undergraduate research projects in this area, in addition to her own work. Both will be attending the 2024 REUF workshop at AIM, where Flagg will lead a research group.

Both are also members of the AIM Research Community (ARC) focusing on these topics, and Flagg is one of the organizers of this ARC. As part of their PRIMES projects, each of them is bringing one of their IEPG-ZF ARC research groups to AIM for a SQuaRE to continue their work. In addition, Flagg is working with AIM Directors to improve infrastructure for the IEPG-ZF ARC, building a sustainable model of a virtual research community that supports continued engagement in research by faculty who have high teaching loads or are unable to travel.

Both Furst and Flagg also emphasize benefits to the students at their colleges in their PRIMES projects, and each has funding to support undergraduate researchers. As part of her project, Furst is in discussion with AIM Director of Programs David Farmer about AIM's Open Textbook Initiative, as the cost of texts is a serious barrier to college attendance for many historically underrepresented and first-generation students. She also intends to work with Farmer and AIM Deputy Director Michelle Manes to develop a mentorship model for future PRIMES applicants. Furst says, "I am excited to have such a wonderful partnership opportunity with AIM and am extremely grateful to AIM, the NSF, and Fort Lewis College." ■

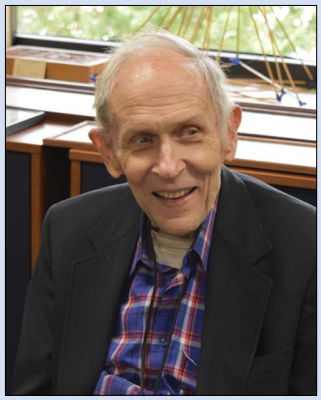
— Leslie Hogben

The 2023 Alexanderson Award

Higher uniformity of bounded multiplicative functions

The recipients of the fifth Alexanderson Award are Kaisa Matomäki, Maksym Radziwill, Terence Tao, Joni Teräväinen, and Tamar Ziegler for their paper "Higher uniformity of bounded multiplicative functions in short intervals on average" published in the *Annals of Mathematics* in 2023. The award will be given at the Joint Prize Ceremony at the 2024 Joint Mathematics Meeting in San Francisco. Joni Teräväinen will deliver the Alexanderson Award Lecture "Uniformity of the Möbius Function in Short Intervals" on Thursday, Jan. 4, 2024, from 11 a.m. – 12 p.m. All five awardees are co-organizing the AIM Special Session on "Multiplicative Number Theory and Additive Combinatorics" on Wednesday, Jan. 3, 2024, from 8 a.m. – 12 p.m. and on Friday, Jan. 5, 2024, from 8 a.m. – 12 p.m.

The Alexanderson Award recognizes an outstanding scholarly article arising from research activities sponsored by the American Institute of Mathematics and published within the past three years. The award was established in 2018 to honor the contributions of Gerald Alexanderson, Professor of Mathematics at Santa Clara University and founding chair of AIM's Board of Trustees. As its first chair, Jerry provided the stewardship that has distinguished AIM as an international center for mathematical research with a commitment to productive and creative collaboration.



This paper, which originated from a working group at the AIM workshop "Sarnak's conjecture" in December 2018, focuses on the local Fourier uniformity conjecture for bounded multiplicative functions. The underlying principle behind the work is the belief that the additive and multiplicative structures of the integers are independent.

Rephrasing that idea more precisely involves the Liouville function λ. If you factor an integer into primes, $n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$, then

$$\lambda(n) = (-1)^{e_1+e_2+\dots+e_k}.$$

In other words, $\lambda(n)$ is 1 if n has an even number of prime factors, and -1 if n has an odd number of prime factors. The goal is to prove that the Liouville function is uncorrelated with a large class of sequences which arise in number theory, and also that it is uncorrelated with any nontrivial transformation of itself.

The pseudorandomness principle says that the Liouville function should behave like a random function $\mathbb{N} \to \{-1,1\}$. That is, $\lambda(n)$ should take on the values 1 and -1 equally often, and furthermore, there should be no patterns in the sequence of 1s and -1s. One way to measure how close $\lambda(n)$ is to random, is to look at the average

$$\langle \lambda(n) \rangle := \frac{1}{X} \sum_{n \le X} \lambda(n).$$

The prime number theorem is equivalent to the statement that $\langle \lambda(n) \rangle = o(1)$ as $X \to \infty$. That statement is fairly weak: It says there is a lack of correlation between values of $\lambda(n)$ over long intervals $[1,X]$. The Riemann Hypothesis (RH) is equivalent to the statement that $\langle \lambda(n) \rangle = O_\epsilon(X^{-1/2+\epsilon})$ for any $\epsilon > 0$. Thus, RH asserts there is a lack of correlation over the much shorter interval $[1, X^{1/2+\epsilon}]$.

The conjectured randomness in $\lambda(n)$ is expressed in another way by Chowla's conjecture (related to the twin prime conjecture) which says that for distinct $h_1, h_2, \dots, h_k$, we have

$$\langle \lambda(n+h_1)\, \lambda(n+h_1) \dots \lambda(n+h_k) \rangle = o(1).$$

In essence, the conjecture asserts that the Liouville function is uncorrelated with any nontrivial translation of itself.

Another consequence of the conjectured randomness of $\lambda(n)$ is that every possible length k sign pattern, $(\pm 1, \ldots, \pm 1)$, would occur infinitely many times as a value of $(\lambda(n+1), \lambda(n+2) \ldots \lambda(n+k))$. In other words, if we let $s_\lambda(k)$ be the number of length k patterns which actually occur infinitely many times, then $s_k(\lambda) = 2^k$. A weak version of Sarnak's conjecture says that λ is not deterministic; that is, $s_\lambda(k) \gg e^{ck}$ for some $c > 0$.

In recent years it has emerged that Chowla's conjecture and Sarnak's conjecture are deeply related, and a large amount of progress has been made on both in tandem. The award-winning paper represents progress towards these two conjectures by proving, among other things, that

$$\tilde{s}_\lambda(k) \gg_A k^A,$$

where $\tilde{s}_\lambda$ counts the number of length-k patterns that occur at least once.

Some details of the mathematics in the paper and its consequences can be found on Terry Tao's blog post: https://tinyurl.com/HigherUniformity. ■

— David Farmer and Michelle Manes

Hello, Goodbye
Changes on the AIM Staff

Estelle Basor retired as AIM's Deputy Director at the end of 2022. She continued to help out with in-person events through June 2023, when AIM moved to Pasadena. Basor began working at AIM in 2008 after retiring from Cal Poly, San Luis Obispo, where she had been on the faculty since 1976. She has held visiting positions at Bryn Mawr and UC Santa Cruz. She received her Ph.D. degree in 1975 from UCSC under the direction of Harold Widom with a dissertation on asymptotics of Toeplitz determinants for singular symbols. Her research interests are in operator theory (especially Toeplitz and Hankel operators) and random matrix theory. At the Newton Institute in 2004, she met Brian Conrey during a semester program on random matrix theory and number theory, and that chance meeting led to her coming to AIM in 2008. Basor was born and raised in Watsonville in the Central Coast region of California. Her family was involved in the apple business, and she has retained an interest in and connection to agriculture ever since. She plans to continue her active research career and to remain involved in farming issues in the Pajaro Valley.

Michelle Manes joined AIM as Deputy Director on July 1, 2023. Prior to joining AIM, Manes spent 15 years as a faculty member in the Department of Mathematics at the University of Hawai'i at Mānoa, conducting research in number theory and arithmetic dynamics, supervising graduate students, and contributing to the teaching mission of the department especially through training preservice elementary teachers. From 2018 to 2021, Manes was a Program Officer in the Division of Mathematical Sciences at the National Science Foundation. She currently serves as the Associate Secretary of the Western Section of the American Mathematical Society and as an Editor-in-Chief for *La Matematica*, the flagship journal of the Association for Women in Mathematics. She is thrilled to move back to Southern California (where she both grew up and had a postdoctoral position) and to join AIM in its core mission of supporting collaborative mathematics and broadening participation.

Claudia Rodriguez-Solorio joined AIM as our Community Outreach and Program Evaluation Specialist in August 2023. She holds a Ph.D. in Public Policy Analysis from the Pardee RAND Graduate School, a Master of Science in Applied Mathematics for the Life and Social Sciences, and a Bachelor of Science in Mathematics from Arizona State University. Before joining AIM, Rodriguez-Solorio specialized in education policy research, focusing on retention and recruitment strategies for historically underrepresented minority students in STEM fields. She also spearheaded a successful initiative to promote STEM disciplines among minority students in her community, which led to an improvement in high school students' performance on state math exams. Previously, Rodriguez-Solorio was the Academic Supervisor for the Joaquin Bustoz Math and Science Honors Program at Arizona State University. In that role, she trained academic staff, developed a supplementary curriculum, and ensured student success. Her interests include program evaluation, math education policy, development of math identity, mathematics education, and mathematical and statistical modeling, as well as quantitative and qualitative research methodologies. She looks forward to contributing her expertise and experience to the outreach initiatives at AIM.

David Crombecque will join AIM as our Associate Director of Special Projects in January 2024, but he has already begun helping connect AIM with outreach activities in Southern California. Crombecque has spent 11 years as a faculty in the Mathematics Department at the University of Southern California, where he is still currently teaching.

Crombecque's interests have focused on broadening participation in mathematics, in particular for people coming from historically marginalized communities. He developed Summer Programs in Mathematics at USC for first-generation students. He is one of the co-directors of the LA Math Teachers Circle and the project leader of Math On the Border. He is also a board member of Spectra, the LGBTQ+ association of mathematicians. Crombecque looks forward to joining AIM full time and collaborating with Los Angeles area math educators, math teachers, mathematicians, and the local math community at large. ■

Welcome New Board Members

Scientific Research Board (term beginning in 2023)

Mihnea Popa, Harvard University
Algebraic Geometry

Human Resources Board (terms beginning in 2023)

Folashade B. Agusto, University of Kansas
Applied mathematics, ecology and evolutionary biology

Mary Flagg, University of St. Thomas, Houston, TX
Graph theory, linear algebra, groups and modules

Marissa Kawehi Loving, University of Wisconsin-Madison
Low dimensional topology and mapping class groups

Noelle Sawyer, Southwestern University, Georgetown, TX
Dynamics, geometry, length spectrum rigidity, and thermodynamic formalism

Advisory Board (term beginning in 2024)

Ron Buckmire, Occidental College
Numerical analysis, scholarship of teaching and learning, and mathematical modeling

Growing Positive Math Identities
Supporting Diverse Students and Educators

In September, AIM hosted a group of 55 committed advocates for math education at a workshop entitled "Toward a Shared Vision for Supporting the Mathematics Identities of Students and Educators from Historically Underrepresented Communities." Sponsored by a planning grant from the Eddie Bernice Johnson INCLUDES program of the NSF, the workshop was unique in bringing together individuals from a variety of sectors of the mathematics education ecosystem: K-12 teachers and higher education faculty, mathematicians and mathematics education researchers, and leaders of nonprofits and community organizations providing out-of-school time math engagement opportunities. The workshop was co-organized by planning grant PI Brianna Donaldson, AIM's Director of Special Projects, along with David Crombecque (USC/AIM), Alessandra Pantano (UC Irvine), Roberto Pelayo (UC Irvine), and Robin Wilson (Loyola Marymount University).

This page, from top: Participants mingle before the workshop dinner at El Portal. Shelly Jones gives an opening address about culturally relevant teaching. Opposite page: Claudia Rodriguez-Solorio talks about supporting the mathematical identities of Latinx students.

The goals of the workshop were to build a shared understanding of what we know about supporting the math identities of students and educators from historically underrepresented communities, to contribute toward a document outlining a "strategic plan" for the field, and to seed future collaborations among participants. It was an action-packed weekend, kicking off with a networking session for L.A.-based participants on Friday afternoon and a banquet for everyone at local restaurant El Portal on Friday evening. Saturday and Sunday mornings included several overview talks considering research-based, experiential, and systems-level perspectives. Each day, these overview talks were followed by a session of five-minute "Lightning Talks," during which participants shared key ideas from their equity-focused work in order to spark further conversations.

The Saturday afternoon working group sessions were a major highlight of the workshop. First, participants split into groups to generate questions for further investigation pertaining to the following six topics: active teaching and learning; building identities and belonging; joyful and out-of-school math; mentoring and critical transition points; systems-level approaches; and teacher preparation, development, and leadership. Using the AIM Moderated Problem Session approach, which is employed in every math research workshop hosted at AIM, the groups were able to generate dozens of questions that will eventually be combined into a problem list for the field. Later in the afternoon, participants split into 10 different working groups that took a deeper look at some of these questions. One participant commented, "I can't reiterate enough how great it was to see such a variety of people across the K-16 spectrum. I am glad you gave us so much time to collaborate."

Key themes that emerged from the workshop

included the importance of fostering belonging, considering the cultural relevance of curricula, providing rigorous yet joyful learning opportunities, and finding ways to effect systems-level change. Ongoing work is focusing on producing a publicly available white paper based on the outcomes of the workshop. Several of the working groups are also continuing to collaborate. As one participant remarked, "This workshop both spoke to my heart and has me thinking about tangible strategies to work within the system to attend to identities." ■

— Brianna Donaldson

Call for Proposals

AIM hosts focused workshops in all areas of the mathematical sciences. AIM's focused workshops are distinguished by their emphasis on a specific mathematical goal, such as making progress on a significant unsolved problem, understanding the proof of an important new result, or examining the convergence of two distinct areas of mathematics. AIM SQuAREs allow dedicated groups of four to six mathematicians to pursue an ambitious research program, with three weeklong in-person meetings held over the course of three years. AIM Research Communities are larger collaborative efforts involving at least 40 people, organized around a particular area of mathematics research, with online meetings and activities.

Each year, AIM accepts proposals for Workshops, SQuAREs, and Research Communities. Proposals are short (generally 2-3 pages) and should focus on the mathematical goals of the activity. The proposal submission window is Aug. 1 – Nov. 1 each year, and the proposal submission form is available at https://aimath.org/ during the submission window. The AIM Scientific Board meets in early December to select programs to support for the following year. AIM staff are always willing to answer questions from potential organizers at workshops@aimath.org.

Did You Know?

Every AIM workshop has an open call for applications, and successful applicants are fully funded, including travel and accommodations. Applications are generally due about five months before the workshop takes place. Workshops in 2024 include:

- **Post-quantum group-based cryptography**, April 29-May 3
- **High-dimensional phenomena in discrete analysis**, May 13-17
- **Groups of dynamical origin**, June 3-7
- **Symmetry-breaking of optimal shapes**, June 17-21
- **Formalising algebraic geometry**, June 24-28
- **Scissors congruences, algebraic K-theory and Steinberg modules**, July 8-12
- **Graph Theory: structural properties, labelings, and connections to applications**, July 22-26
- **PDE methods in complex geometry**, August 26-30

You can always check out upcoming workshops on the AIM website: https://aimath.org/workshops/.

Dispatches from Morgan Hill

Shaping the Present and Future of Students

Morgan Hill Math is an outreach program sponsored by the American Institute of Mathematics, which provides free math enrichment activities and opportunities for math competitions to about 300 students each year, who live in or near Morgan Hill, Calif.

Most families are introduced to Morgan Hill Math through Mathletics and MathCounts6. In Fall 2022, our youngest students, fourth and fifth graders enrolled in our eight-week Mathletics program, enjoyed lessons which included learning to play SET, solving logic problems, making a tangram set, an introduction to modular arithmetic and building a quilt square, discovering prime numbers, and practicing problem-solving strategies such as working backwards and drawing pictures. Students in MathCounts6 were introduced to more advanced problem-solving concepts including counting in other bases, problem posing, proportions, and combinatorics, all skills that are necessary for success in math competitions.

"(My child) had a really great time at Mathletics last year… I wanted to thank you for instilling a love of math and problem solving in him and all the other kids. The program that you are running is fantastic." (Mathletics Parent)

Local seventh and eighth graders were invited to participate in weekly MATHCOUNTS trainings throughout the fall of 2022. Over 50 students, from 6 local schools, sharpened their skills in factorials, word problems, triangles, probability, permutations and combinations, functions, algebra, and geometry. Happily, MATHCOUNTS returned to in-person competitions for the 2022-2023 season. Over 40 students attended our chapter competition, and 10 qualified to move onto the Northern California State Competition held at the University of the Pacific in Stockton.

"Thank you so much for teaching me for the past five years. You helped me discover my love for math and inspired me to pursue a career in math." (8th grade program participant)

All interested students from Morgan Hill Math programs, 8th grade and below, were invited to participate in the Math Olympiad for Elementary and Middle School (MOEMS). Over 70 kids were challenged to strengthen their problem-solving skills through a monthly five-question test. In the middle school division, eighth grader Andrew Zhang achieved a near perfect score (24). In the elementary school division, fourth grader Aarya Dhane outscored all the other fourth through sixth graders in Morgan Hill Math with a score of 23. In May 2023, we rented the Morgan Hill Playhouse for the Math Olympiad Award Ceremony. The kids got to come up on stage to receive their certificates and awards! It was an amazing experience and really celebrated the kids who continue to challenge themselves through such a difficult math contest.

Math Olympiad Winners, 2022/2023 season

Coyote Valley Chapter Competitors, February 2023

Mathletics Class and activity, fall of 2022

MATHCOUNTS State Competitors

"Throughout the years. I have learned so much from you and your teaching has made me into the student I am today." (Former participant who is now attending Stanford)

Our middle and high school Mathletes participated in several other competitions this year. Thirty-six students took the AMC8, an MAA competition for students in eighth grade and below. Three students, Rui Qian Khor (8th grade), Mason Nishimura (7th grade), and Andrew Zhang (8th grade), scored in the top 5 percent nationwide, earning them a spot on the Honor Roll. High school students took the AMC10 or AMC12 exams. For the sixth year in a row, a Morgan Hill Math student (Ethan Fang, 12th grade) qualified for the AIME, the American Invitational Mathematics Exam, which is the first in a series of examinations that culminate with the International Mathematical Olympiad (IMO). Two teams also competed in the Purple Comet! Math Competition.

"Thank you so much for all your dedication and hard work for the kids. You truly go above and beyond and have instilled such a sense of joy for math for my kids!" (MH Math parent)

The Morgan Hill Student Math Circle for elementary and middle school students continued throughout the year. We met every Monday over Zoom to explore fun math activities, mostly drawn from the Math Circles and JRMF webpages. The Circle has grown this year to include students from second through eighth grades. As an online circle, families are registering from all over the U.S. and Canada!

Making connections with students and families is what makes Morgan Hill Math so rewarding. Watching that spark light up in a student's eyes when they understand a new idea, and seeing them gain confidence and enjoyment in problem solving: there is just nothing like it. Hearing from them what a difference Morgan Hill Math has made in their journey is amazing.

"Thanks again for offering this amazing program to the students.… We are thankful & blessed to have you in the community." (MH Math parent) ■

— *Kelley Barnes*

Quilt Reflection activity from Mathletics.

Resolving Our Differences

Syzygies and Mirror Symmetry

Each year, AIM reserves a couple of weeks to accommodate "Hot Topics" workshops on questions of emerging interest that can be arranged on (relatively) short notice, such as the "Syzygies and mirror symmetry" workshop held in September 2023.

Mirror symmetry is a deep relationship between symplectic topology and algebraic geometry originally suggested by string theory. The homological mirror symmetry conjecture, proposed by Kontsevich, gives a geometric interpretation of mirror symmetry as an equivalence of categories. The Fukaya category, which captures the Lagrangian intersection theory of a symplectic manifold, is related to the derived category of coherent sheaves on a mirror space. Mirror symmetry has had a profound impact on symplectic topology and algebraic geometry and led to many fruitful interactions between these two fields.

More classically, the field of algebraic geometry is built upon the principle that the geometry of solution sets of polynomial equations can be understood through commutative algebra. Expressing the geometry in terms of a polynomial ring also allows for computer algebra tools to be brought to bear on computational problems. This perspective has been thoroughly developed for varieties in projective space where *syzygies* of the defining ideals of subvarieties have been shown to capture many geometric features of the subvariety.

Vector spaces over a ring always have a basis. Finitely-generated modules over a ring have generating sets, but generally do not have bases. A *syzygy*, besides being a great Scrabble word, is a nontrivial relation among the generators of an R-module. For example, if $m_1, m_2, \ldots, m_n$ are generators of a module M over a commutative ring R, then a *syzygy* of M is an element $(r_1, r_2, \ldots, r_n) \in R^n$ such that

$$r_1 m_1 + r_2 m_2 + \ldots + r_n m_n = 0.$$

The set of all syzygies (relative to the given generating set) is a submodule of R^n, called the module of syzygies, and it is precisely the kernel of the map $R^n \to M$ taking the standard basis of R^n to the generators of M. Of course, one could then ask about the syzygies of the module of syzygies, and so on and so on. Continuing this process until you get a free module (one that actually does have a basis, so no nontrivial syzygies) is called a *resolution* of the module M. Hilbert's syzygy theorem states that, if M is a finitely generated module over a polynomial ring in n variables over a field, then the nth syzygy module of M is always a free module. In other words, the length of the resolution is bounded by the dimension of the module.

Participants in the "Syzygies and mirror symmetry" workshop.

Until recently, mirror symmetry has had few direct applications to commutative algebra. In March 2023, Andrew Hanlon, Jeff Hicks, and Oleg Lazarev posted the paper "Resolutions of toric subvarieties by line bundles and applications" on the arXiv. The paper, using techniques from symplectic geometry and mirror symmetry, resolved a major open problem in the world of commutative algebra, a virtual analogue of Hilbert's syzygy theorem for smooth projective toric varieties conjectured by Berkesch, Erman, and Smith over a decade ago. The techniques in the paper opened up exciting new possibilities at the intersection of symplectic geometry and commutative algebra. In proposing a "Hot Topics" workshop on these ideas, Erman wrote, "Basically everyone I know of who works on multigraded commutative algebra is now diving into this paper. There is a big gap between the authors' viewpoint (largely symplectic geometry and mirror symmetry) and our viewpoint (commutative algebra and classical algebraic geometry), but that makes the conversations

that much more exciting. A weeklong meeting with some of these folks could have a really major effect on both fields."

From September 5 to 8, 2023, over 30 mathematicians gathered at AIM for the workshop "Syzygies and mirror symmetry," organized by Daniel Erman and Andrew Hanlon, to explore where these ideas would lead. The workshop featured talks meant to introduce the main ideas and problems of interest in each field. Working groups were formed during the week, some of them working to build and understand a kind of "dictionary" between mirror symmetry and commutative algebra and others working on open problems in each field using new ideas they were learning. Many of the groups continue to collaborate, and we can't wait to see what comes from this exciting workshop in the months and years ahead. ■

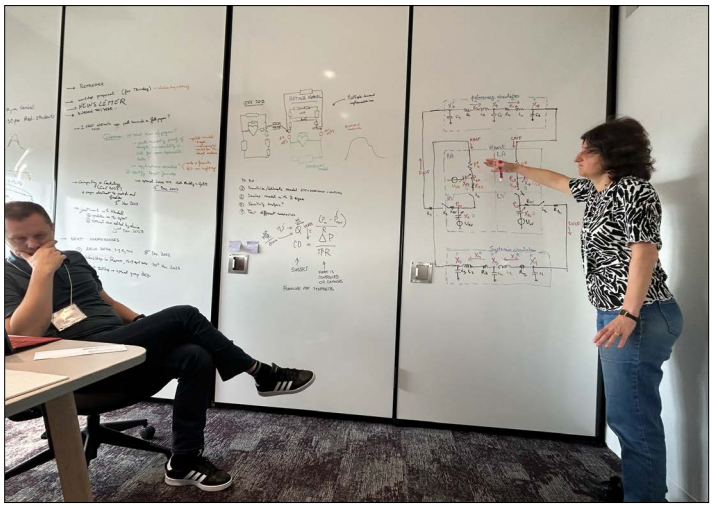
— *Michelle Manes*

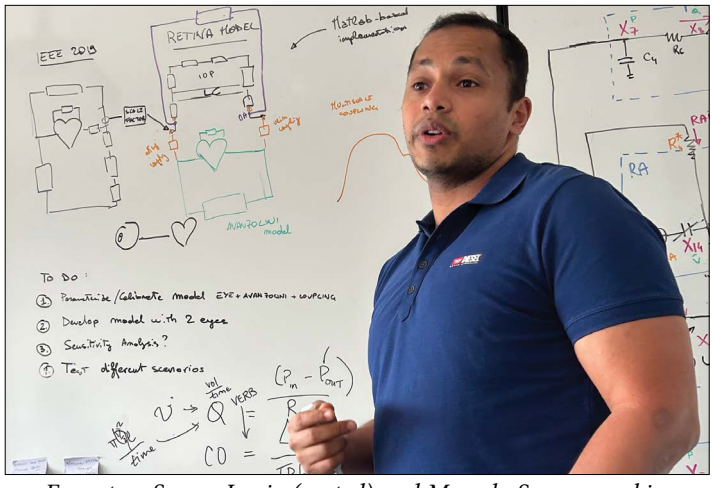
A SQuaRE With a Lot of Heart

Exploring an Intersection of Math and Cardiology

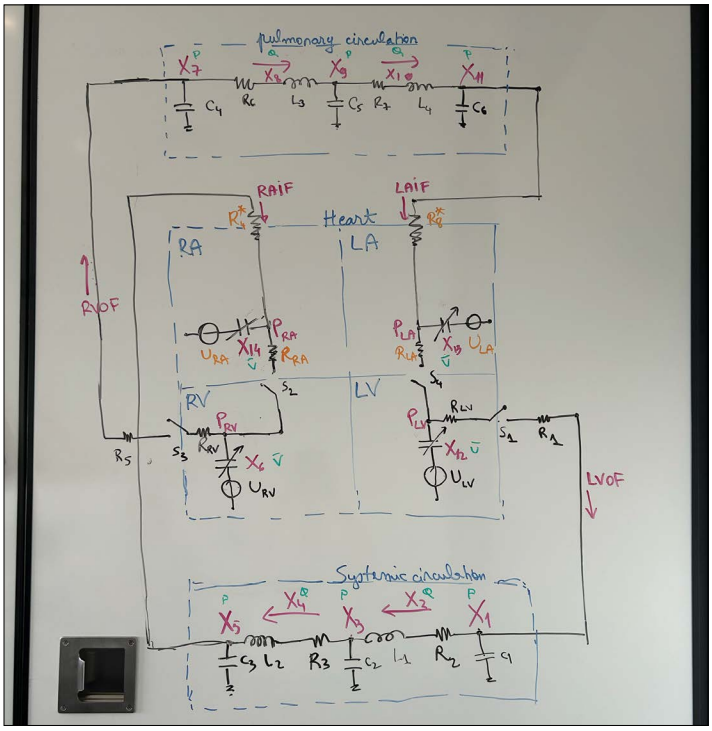
The AIM SQuaRE "Modeling the relationship between cardiovascular function and ballistocardiogram" recently had its second week-long in-person meeting, in addition to one meeting that was held virtually during the pandemic. The SQuaRE consists of a diverse group of scientists with respect to nationalities, career stages, training, expertise, and interests, who straddle the worlds of mathematics, physics, engineering, and biology. The team includes Virginia Huxley (University of Missouri - Columbia), an emeritus professor of cardiovascular physiology; Sergey Lapin (Washington State University), a professor of data analytics; Marcela Szopos (Université Paris Cité), a professor of applied mathematics; Lorenzo Sala (National Research Institute for Agriculture, Food and the Environment), a junior research scientist in biomathematics; and Mohamed Zaid (University of Maine), a Ph.D. student in biomedical engineering. Giovanna Guidoboni (University of Maine), a Dean of Engineering and professor of electrical and computer engineering, joined the group remotely.

Most people are familiar with the electrocardiogram (ECG), which is recorded using electrodes placed on the chest to measure the electrical activity generated by each heartbeat. The ECG is used to diagnose heart conditions, including ischemic heart disease and abnormal heart rhythms. Less familiar is the ballistocardiogram (BCG), first discovered in 1877





From top: Sergey Lapin (seated) and Marcela Szopos working on mathematics. Mohamed Zaid in front of the group's work.



From left: Participants in "Modeling the relationship between cardiovascular function and ballistocardiogram" meet at AIM. A model of the cardiovascular system connecting the heart and the eye.

by J. W. Gordon, a signal that captures the mechanical and fluid dynamical properties of the cardiovascular system. The BCG is recorded without restraint, using different sensing modalities such as bed sensors under the mattress, accelerometers under the pillow, and scales. For example, if you lie perfectly still, while holding your breath, your body still moves. The movement is generated by the heart beating, moving blood through your body. While having the advantage of being acquired non-invasively, use of BCG as a clinical monitoring and diagnostic tool is limited by a lack of understanding of the cardiovascular mechanisms responsible for changes in the signal. Computer-aided approaches can help provide a mechanistic and quantitative interpretation of BCG signals, as pioneered by Starr and Noordergraaf in 1967.

The ballistogram was a good starting point for this SQuaRE's collaboration, as this non-invasive assessment of cardiovascular and respiratory function has the potential to detect early signs of systemic cardiovascular, respiratory, and neurological pathologies. The difficulty addressed by the group was that the meaningful clinical data and subsequent interpretations are hidden within a composite of multiple dynamic events. The analyses of the group leveraged the powerful synergy of mathematical modeling, experimental data, clinical insights, and technological advancements.

On the mathematical side, modeling of the BCG involves intricate multiscale and multiphysics components. Deconvolution of this complex signal demands collaboration among experts in ordinary and partial differential equations, fluid-structure interaction, and numerical analysis. During the first stage, the group focused on a reduced modeling approach, based on the analogy between electric and hydraulic networks. The advantage of these new network-based models is that they provide a systemic view, able to capture the overall dynamics of the interwoven physiology of fluids in the eye and in the heart, while maintaining a relatively accessible mathematical complexity and low computational costs.

Mathematical modeling plays a crucial role in unraveling the complexities of systemic diseases, and these models are already serving as virtual labs, elucidating driving mechanisms, quantifying contributions of factors, and testing clinical hypotheses. ■

— Giovanna Guidoboni, Virginia Huxley, Sergey Lapin, Marcela Szopos, Lorenzo Sala, and Mohamed Zaid

A Dynamic Research Community
Collaborative Learning Among Graduate Students

AIM is thrilled to have returned to in-person workshops and SQuaREs, but we're taking the lessons we learned from moving our collaborative activities online to our AIM Research Communities. These Communities are larger than Workshops, with at least 40 participants and often many more, and have a longer time frame of a year or more.

AIM Research Communities are tailored to the particular needs of the group. For example, the Big Ideas in Dynamics Research Community focuses on collaborative learning among graduate students in dynamical systems, specifically smooth dynamics, ergodic theory, and homogeneous dynamics. The goals are to build connections across institutions and to strengthen the reading, research, and collaboration skills of current and recent graduate students.

The Community is organized by Benjamin Call from University of Illinois Chicago and Noelle Sawyer from Southwestern University. The program takes place across one semester and consists of three phases:

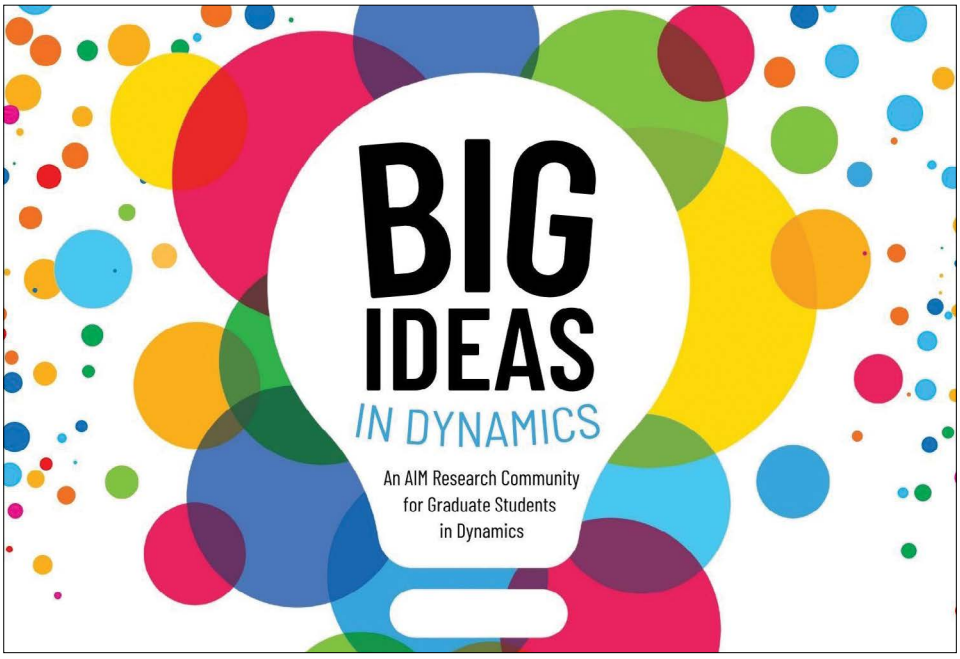
- The program launches with a virtual learning conference, where senior mathematicians give short talks about a paper or idea necessary to understand some of their work.
- Throughout the semester, graduate students meet virtually in small groups to read through one of the presented papers. A mentor, usually a postdoc or senior graduate student who is familiar with the paper, holds regular office hours to support the reading groups.
- A virtual closing conference includes short talks by the reading groups, talks by the mentors on their research, and a problem session to encourage continuing collaboration.

The first semester-long program launched on February 3 and 4, 2023, with talks by Vaughn Climenhaga (Specification and Measures of Maximal Entropy), Osama Khalil (Mixing, Counting and Equidistribution in Lie Groups after Eskin and McMullen), Bryna Kra (Infinite Patterns in Sets of Positive Density: Translating Combinatorics to Dynamics), and Amie Wilkinson (Pathological Foliations). Graduate students then formed groups to dig into some of the foundational papers in the field.

The semester was such a success that the organizers decided to do it all over again. On September 29, 2023, a new group of students joined the Community. The launch event included talks by Alena Erchenko (Flexibility of entropies for surfaces of negative curvature), Giulio Tiozzo (Random walks on weakly hyperbolic groups), and Caglar Uyanik (Length functions on currents and applications to dynamics and counting).

Find out more about this Research Community on their AIM webpage (**https://aimath.org/programs/researchcommunities/bigdynamics/**). You can view the talks and some of the closing event activities at their website (**https://sites.google.com/view/bigideasindynamics/home**). ■

— *Michelle Manes*



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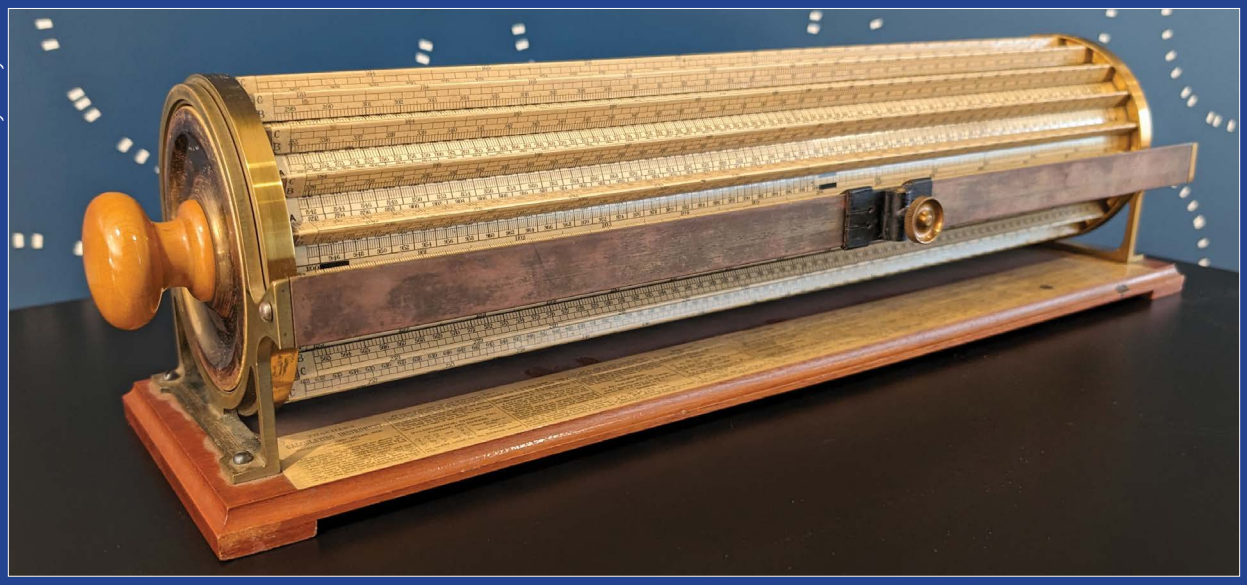
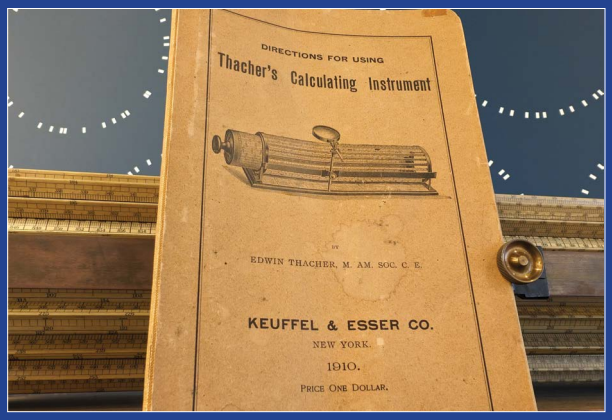
Photos by Terry Busk





A cylindrical slide rule donated to AIM in 2023 by Keith Dennis.