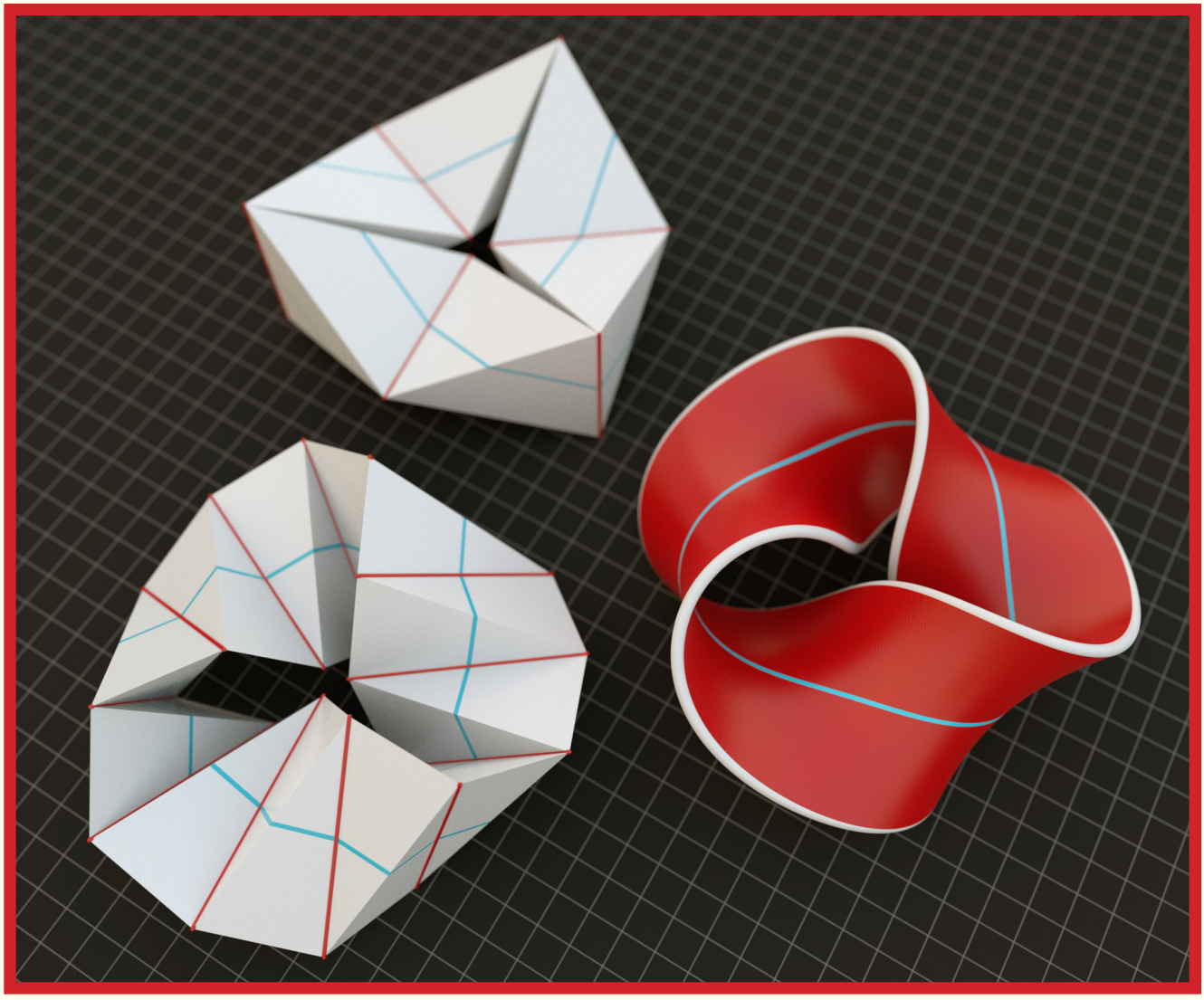
Autumn 2019

AIMatters

Newsletter of the American Institute of Mathematics



Inside:

Letter from the Director

Looking Back at 2018

Greetings from San Jose! We've had an exciting year this year which marked the twenty-fifth anniversary of the founding of AIM on June 28, 1994 (see p. 11).

Especially notable was our celebration of our first Alexanderson Award winners with an event at Santa Clara University featuring the prize winners and their families and a splendid talk by Persi Diaconis (see p. 8). More than 700 mathematicians from around the world spent a week with us this year for a workshop or SQuaRE. Their AIM research projects have resulted in more than 100 papers thus far.

The number of Julia Robinson Mathematics Festivals (JRMF) exceeded 100 for the first time ever! This included a festival that drew thousands of kids and parents as a large part of the National Math Festival and a festival on the last day of the 2019 MathFest in Cincinnati that had close to 200 mathematicians in attendance.

We hosted the Global Math Project 2018 launch; that program has now reached more than five million students across the world. And we have taken under our wing a new program called Math Mondays, which involves parents at elementary schools hosting math games for students over the Monday lunch hour. Conceived by Scott Kim, who contributed to the development of the popular games Rush Hour and Bejeweled, this program has been pilot tested for two years in Burlingame and is now ready for wider circulation. We expect this program to become widespread.

The workshop on Sarnak's Conjecture led to a big mathematical breakthrough that you can read about on page 14 in this newsletter. And thanks to Fry's Electronics we have a new 2600 square foot library space downstairs from our usual meeting space.

Finally, our biggest news is that we received a gift of 1.4 million dollars from the estate of Morris Weisfeld, Professor of Mathematics at Duke University and founder in 1991 of the journal International Mathematics Research Notices (IMRN). IMRN is world renowned for its cutting-edge articles and fast publication time and is a favorite of up-and-coming researchers. We are extremely grateful for this gift from Professor Weisfeld that officially begins the AIM Endowment Fund.

As always, if you are in the Bay Area we invite you to stop in to visit. And in the meantime I hope you enjoy this issue of AIMatters. ■

Brian Conrey

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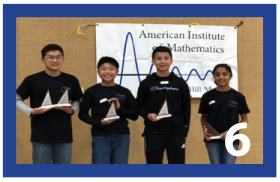

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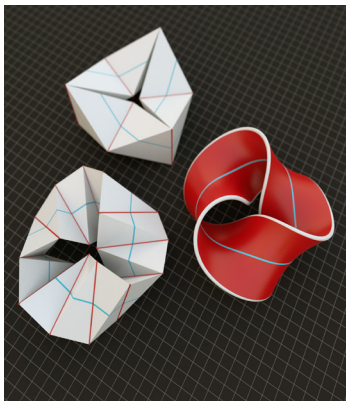


One of our local AIM Math Teachers' Circle presenters last year was Eliot Fried, a professor at the Okinawa Institute of Science and Technology Graduate University in Okinawa, Japan. He kindly shared his constructions of his beautiful kaleidocycles. A kaleidocycle is a three-dimensional ring, composed of a series of linked tetrahedra.

Top: Classical kaleidocycle with six hinges.
Left: Möbius kaleidocycle with twelve hinges. Hinges are shown in red. Möbius kaleidocycles are ring linkages with seven or more hinges. Like the classical six-hinged kaleidocycle, each Möbius kaleidocycle has just one internal degree of freedom.
Right: The three half-twist symmetric Möbius band arising as the limit shape of a Möbius kaleidocycle with infinitely many hinges. This surface is ruled, has pointwise negative Gaussian curvature, and its midline is a curve of constant torsion.

To see more or make these yourself, please see:
http://hannes.home.oist.jp/projects/babylon/kaleidocycles.html, and
http://hannes.home.oist.jp/kaleidocycles/Kaleidocycle_instructions.pdf.

Julia Robinson Math Festivals

Developing School Partnerships

On Saturday, May 11, 2019, about 100 students and their families gathered at Gwinn Elementary School in San Martin, CA, for a Julia Robinson Mathematics Festival (JRMF).

The festivals are locally organized events intended to inspire K-12 students to explore the richness and beauty of mathematics through problem-solving. Founded in 2007 by Silicon Valley native and math puzzle enthusiast Nancy Blachman, JRMFs are collaborative, community-friendly mathematics festivals intended to serve as an alternative to competitions for getting students—especially girls and students of color—and their surrounding communities engaged in mathematics. To inspire participants to persist in mathematics, Ms. Blachman named the festivals after mathematician Julia Robinson, who was one of the solvers of Hilbert's tenth problem and who overcame many obstacles to achieve a highly successful career as a woman in mathematics.

Left, above, and facing: Children and adults revel in mathematical puzzles and games galore at various JRMF functions.

The Julia Robinson Festival at Gwinn School was special because it marked the beginning of a collaborative partnership between the American Institute of Mathematics (AIM) and the Santa Clara County Office of Education (SCCOE), which serves nearly 300,000 students in the most populous county in the San Francisco Bay Area. The partnership focuses on bringing the Julia Robinson Festivals to Title I schools in Santa Clara County. Under AIM's guidance, Math Teachers' Circles (MTCs) will play a crucial role in the partnership. Specifically, during MTC meetings, teachers will be introduced to festival problem-solving activities so that they can help run festivals and use the activities in their classrooms.

Brian Conrey, Executive Director of AIM said, "We envision these JRMFs, combined with MTCs and other synergistic programs like Math Mondays, as forming the basis for strong, school-based mathematical communities. We hope that this partnership with SCCOE can grow into a model that is of interest to other MTCs looking to connect more directly with schools." ■

- Brianna Donaldson

JRMF activities are highly accessible, yet provide opportunities for rich investigation. One activity used in the Gwinn JRMF was "Puppies and Kittens."

PUPPIES AND KITTENS

A number of puppies and another number of kittens are in two pens. Two players take turns making one of three possible moves: taking any number of puppies, or any number of kittens, or the same number of each. So, for example, if there are 8 puppies and 6 kittens, a player can take 4 puppies in one turn, or 2 kittens, or 3 puppies and 3 kittens.

One player decides the starting number of puppies and kittens and the other player decides who goes first. The winner is the player who takes the last animal remaining.

Challenges

1. For any starting number of puppies and kittens, is there an optimal strategy so one player is guaranteed to win?
2. How might a player visualize the winning strategy for this game?

- Mark Saul

For more information and additional JRMF activities, please visit https://www.jrmf.org/.

Dispatches from Morgan Hill
Mathletes and More

Morgan Hill Math continues to grow and provide new opportunities for students in the Morgan Hill community who are interested in exploring math outside of the classroom. Morgan Hill Math is an outreach program sponsored by the American Institute of Mathematics (AIM) that provides free math enrichment activities to about 300 students each year.

This year we held two Julia Robinson Mathematics Festivals (JRMFs) in local elementary schools, at Barrett Elementary School in March, and in May we worked with the Santa Clara County Office of Education to offer a festival at San Martin/Gwinn Elementary School. (For more information about this event, see the article on p. 4.) Both of these are Title I schools. A Julia Robinson Mathematics Festival is an event at which students play with mathematics. Students choose among a dozen tables offering problem sets, games, activities, or puzzles with mathematical themes. Dozens of kids and their parents enjoyed solving puzzles, playing games such as SET or Queen's Move, and moving through giant floor mats of mazes and puzzles. We will definitely be holding more of these engaging events in Morgan Hill!

Most families are introduced to Morgan Hill Math through Mathletics and MathCounts6. Working in the spring with the Morgan Hill Unified School District, advanced math students in third through fifth grade are identified and invited to try out for a spot in the fall math enrichment programs. Last fall, our youngest students, fourth and fifth graders, enrolled in our eight-week Mathletics program, enjoyed lessons which included learning to play SET, building Tangrams, solving logic problems, working backwards to solve problems, and making quilts using modular arithmetic. Students in MathCounts6 were introduced to more advanced problem-solving concepts including proportions, permutations and combinations; all skills that are necessary for successfully competing in the MATHCOUNTS competition series.

Competitors from the MATHCOUNTS Coyote Valley Chapter Competition.

The cornerstone of the Morgan Hill Math program has always been training for and competing in MATHCOUNTS. This Competition Series has four levels of competition: school, chapter, state, and national. This year, over 60 students, from seven local schools, trained with me once a week, from September through December, sharpening their skills in counting, series and sequences, probability, permutations and combinations, functions, algebra, and geometry. In January, 45 students came together to prepare as a group for the chapter competition. High school students who had previously competed in MATHCOUNTS volunteered to help train these mathletes.

Morgan Hill students compete in the Coyote Valley Chapter, held in Morgan Hill. For the third year in a row, the team from Martin Murphy Middle School won the team round! Second place went to the team from Oakwood, who barely edged out Charter School of Morgan Hill. The eight students from the top two teams, as well as a student from Jackson Academy of Math and Music and another Murphy student, qualified to compete in the Northern California MATHCOUNTS State Competition at Stanford.

Enthusiastic students from these fourth- through eighth-grade programs were invited to participate in the Mathematical Olympiads for Elementary and Middle Schools. We had so many middle school students eager to compete this year that we had to form an additional class just for the Jackson Academy of Math and Music seventh- and eighth-grade students. From November to March, over 110 kids were challenged to strengthen their problem-solving skills in this once-a-month, five-question test. One Morgan Hill student, Ethan Fang, won the George Lenchner Medallion for achieving a perfect score in the Middle School division! Only 0.5% of the nearly 100,000 participants received this award.

The Morgan Hill Math Teachers' Circle (MTC) is still growing. Local math teachers in any grade can attend monthly meetings to explore rich math problems, rediscover the joy in learning math, and network with fellow teachers and mathematicians. Meetings were held on the last Wednesday of the month, at the Morgan Hill Community and Cultural Center. We held six meetings over the course of the school year where we explored SOMA Cubes, Venn Diagrams, and JRMF activities as well as classic MTC activities such as Grid Power, Mental Math, and Derangements. Two of our participants elected to go to the AIM MTC Summer Immersion Workshop and are eager to bring their colleagues to next year's meetings. The Morgan Hill Math Teachers' Circle is part of the Bay Area Teachers and Mathematicians (BATMath) Network.

Our middle school Mathletes participated in several other competitions this year. All students involved in the weekly MATHCOUNTS training classes, as well as interested fifth and sixth graders, took the AMC8, an MAA competition for students in eighth grade and below. Many of them also challenged themselves with the AMC10 exam. By scoring in the top 2.5% in the United States and Canada on the AMC10, ninth-grader Neil Shah qualified to take the AIME, the American Invitational Mathematics Exam, which is the first in a series of examinations that culminate with the International Mathematical Olympiad (IMO).

Two local schools competed in the weekly Math Madness competition: Sobrato High School and Martin Murphy Middle School. As a joint initiative between American Mathematics Competitions (AMC) and AreteLabs, Math Madness is emerging as one of the premier math competition events in the United States. Sobrato did very well, making it to the quarter finals in their division, ultimately losing to the division winners. Amazingly, Ethen Fang, from Murphy, finished in twelfth place out of all the competing middle school students nationwide!

Lastly, the Math Club from Sobrato held a Math Mardi Gras for students who attended the after-school YMCA program at P. A. Walsh Elementary School. These high schoolers had a wonderful time working with the third graders and leading them through math activities. The youngsters enjoyed exploring the games and earning Mardi Gras beads for their accomplishments. Live Oak is planning to hold a similar event at Barrett Elementary School in December. It is so heartwarming to see these Morgan Hill Math alumni giving back to their community and spreading the joy of math to Morgan Hill's future mathletes! ■

-Kelley Barnes

The Alexanderson Award
2018 and 2019 Recipients

The first annual Alexanderson Award was presented to Alexei Borodin, Ivan Corwin, and Patrik Ferrari on December 12, 2018, in the Recital Hall at Santa Clara University. The award, which is given in honor of Gerald Alexanderson, Emeritus Professor of Mathematics at Santa Clara University and founding chair of AIM's Board of Trustees, recognized their article "Free energy fluctuations for directed polymers in random media in 1+1 dimensions," published in *Communications in Pure and Applied Mathematics*, 67 (2014), 1129-1214. This work began during the October 2011 AIM workshop, "The Kardar-Parisi-Zhang equation and universality class."

Following the award ceremony, the Inaugural Alexanderson Award Lecture was delivered by Persi Diaconis of Stanford University. This talk, "Universality and the Taming of Randomness," was focused on how to make sense of randomness ("Why does the other lane always move faster than the one I chose?"), and how to make predictions ("How high should I make the floodgates to keep things safe?"). Diaconis described some new approaches to answer these questions and explained how the new laws of chance help understand how to harness chance. He also described how the new laws discovered by Borodin, Corwin, and Ferrari fit into the laws of uncertainty. The talk generated many interesting questions, and the discussions continued at a lovely reception honoring the award winners and Gerald Alexanderson.

A video of the lecture and award ceremony is available on the AIM website, along with more detailed information about the 2018 winning paper. ■

- Estelle Basor

Left: Brian Conrey, Ivan Corwin, Alexei Borodin, Keith Devlin (Emcee), Patrik Ferrari, and Persi Diaconis. Right: Patrik Ferrari, Alexei Borodin, Ivan Corwin, and Amir Dembo.

In August 2019, the American Institute of Mathematics (AIM) announced the second annual Alexanderson Award. The Alexanderson Award recognizes outstanding research articles arising from AIM research activities that have been published within the past three years. Receiving this year's award are Paul Bruillard, Siu-Hung Ng, Eric C. Rowell, and Zhenghan Wang for their paper "Rank-finiteness for modular categories" published in the *Journal of the American Mathematics Society* in 2016. The award will be presented at the annual Alexanderson Award Ceremony and Lecture on October 4, 2019. The lecture will be given by Jordan Ellenberg, the engaging author of the book *How Not to Be Wrong*, in the Recital Hall of Santa Clara University at 7:30 p.m.

Understanding and organizing the basic building blocks of mathematics and physics is one of the most natural of scientific endeavors. We do this with the elements in our periodic table, with descriptions of elementary particles, such as quarks, photons, etc., and we do this with many mathematical structures. Perhaps the simplest is the fact that every integer is a product of primes.

The authors of the 2019 Alexander Award paper studied the properties of mathematical structures called modular tensor categories (MTCs). These structures are models for topological phases of matter which occur at near-zero temperatures. The authors developed a way of indexing the MTCs by a non-negative integer called the rank, and within each rank proved that there were only a finite number of such objects.

To see how one might prove such a result, here is a simple version of the technique. If one considers the equation $a^2 + b^2 = c^2$ and asks for all relatively prime integer solutions, there are infinitely many triples, called Pythagorean triples. But if one requires that a, b, and c only have prime factors from a finite set of primes, then there are only finitely many solutions. This is not obvious, and it is rather difficult to prove. Something similar was carried out in the winning paper by making use of concepts analogous to the prime numbers and integers.

The results in the paper are a wonderful example of the interplay of mathematics and physics and understanding the physical world through the elegance of mathematics. For a complete description, please visit **https://aimath.org/alexanderson-award-2019/**. ■

- Estelle Basor

Gerald Alexanderson was a member of the Santa Clara University faculty beginning in 1958 until his retirement in 2018. During that time, he was Chair of the Mathematics department for 35 years and a member of the Faculty Senate Council. For thirty eight years he held the endowed Valeriote Professorship of Science Chair. He is author of more than a dozen books, including textbooks in abstract algebra, and discrete and combinatorial mathematics. Alexanderson was the first recipient of Santa Clara University's Bayma Award for Scholarship, and he received the Special Appreciation Award from the Dean of Arts and Sciences as well as the Special Recognition Award for Teaching, Research, and Service from the President of the University.

Alexanderson's influence has extended to the national level, where he has played a leading and lasting role in the Mathematical Association of America (MAA). His contributions to the MAA have spanned more than 50 committees and 24 years on the Board of Governors, encompassing Secretary, Vice-President, and President of the Association and Editor of *Mathematics Magazine*. Results of this work include the remodeling of the MAA Carriage House in Washington, D.C., into its Mathematical Sciences Conference Center. In this time, Jerry served on the Science Policy Committee of the American Mathematical Society (AMS) and was a consultant to the Editorial Board for the Bulletin of the AMS. In testament to his expansive record, Alexanderson received the MAA's most prestigious award for distinguished service to Mathematics, the Yueh-Gin Gung and Dr. Charles Y. Hu Award.

Global Math Project 2018
Kick-off Symposium and Party

The kick-off event for the 2018 Global Math Week was a day-long symposium at Santa Clara University followed by a reception at The Tech Interactive in San Jose. The goal of the Global Math Project (GMP) is to engage students and teachers around the world in thinking and talking about the same piece of mathematics during one special week each year to create a forum for the global celebration of creative mathematical thinking.

The symposium took place October 6 in the beautiful Charney Hall, where a series of intriguing talks were given by mathematicians who spoke passionately about the joy of math. The speakers included Jo Boaler, a professor of mathematics education at Stanford University; Francis Su, a mathematics professor at Harvey Mudd College; and James Tanton, one of the founders of GMP and currently the Mathematician-at-Large for the MAA. Following opening remarks from Brian Conrey and Keith Devlin ("The Math Guy" on National Public Radio), Tanton gave a talk on Exploding Dots, a refreshing and powerful mathematical approach to arithmetic and algebra and one of the main activities of the Global Math Week.

Later Aileen Rizo, a professor at Fresno Pacific University, spoke of what a mathematician "looks like," explaining that her daughter had drawn a picture of an Albert Einstein look-alike when asked that very question. She went on to talk about how this stereotype needs to change to reflect the greater population of people who simply love math, especially women and people of color.

Up next was Jo Boaler who showed examples of timed tests that are given out to elementary students to practice their addition and multiplication. She explained that the usual reaction to the image is one of dread or stress and emphasized that mathematics education needs reforming to excite students about mathematics.

Participants enjoy the kick-off celebration for Global Math Week 2018.

After the symposium ended, the event transformed into a Julia Robinson Mathematics Festival (JRMF) outside the hall where kids came to play enticing math games and puzzles.

The day culminated with a party to celebrate the coming week of math. The location in the Tech allowed attendees to wander about the exhibits and even interact with them through their special "Tech Tag." The partygoers danced the electric slide while listening to classics spun by the DJ.

Suffice it to say that Global Math Week 2018 started off with a blast, which continued on through the week as the number of participants increased to over five million people from all over the world! This year's Global Math Week begins October 10, and more information can be found at **globalmathproject.org**. ■

- Sonya Kohli

A "Perfect" Anniversary

Celebrating 25 Years of AIM

June 28, 2019, marked the 25th anniversary of the founding of AIM by businessman and math enthusiast John Fry. In 2002 AIM became part of the National Science Foundation (NSF) Mathematical Sciences Institutes program. We hosted two workshops in 2002 followed by thirteen in 2003. In 2007 we added the program called SQuaREs for small research groups. That year we hosted two SQuaRE groups and six the following year. In 2019, we will host a total of 20 workshops and 64 SQuaRE groups. Almost every week at AIM there is some activity, either a workshop, multiple SQuaREs, or an outreach program.

The number of outreach activities has also greatly increased. In 2008 AIM began organizing Math Teachers' Circles (MTC) with an initial group of six. Now there are 130 in 40 states. In 2013 AIM became the sponsor of The Julia Robinson Mathematics Festival (JRMF) program. The following year there were 11 festivals, and we expect over 100 in 2019 for the second consecutive year. Festivals have been held in 26 states and 12 foreign countries.

On June 28, we celebrated our anniversary with champagne and cupcakes along with staff and some of our SQuaREs participants. ■

- Estelle Basor

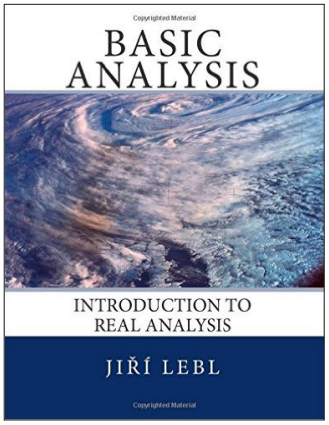
Perfect Numbers

The date of our anniversary, June 28, or 6/28, has an interesting mathematical significance. Both six and 28 are perfect numbers; that is, they are the sum of their divisors, excluding themselves, $1 + 2 + 3 = 6$, and $1 + 2 + 4 + 7 + 14 = 28$. Can you find the next one? It is not known whether any odd perfect numbers exist, or whether there are infinitely many perfect numbers.

AIM staff and SQuaRE participants raise their glasses to honor AIM'S 25th anniversary.

Open Access
AIM's Textbook Project

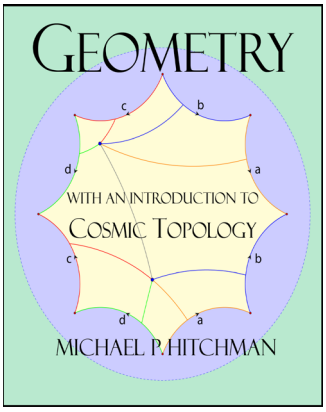
Are you looking for a textbook for a math course? Would you like to find one that is free in a PDF version, free for an interactive online version, and modestly priced at around $25 for a printed version? Or would you rather pay well over $100 to $200 for a printed copy and no PDF or online version?



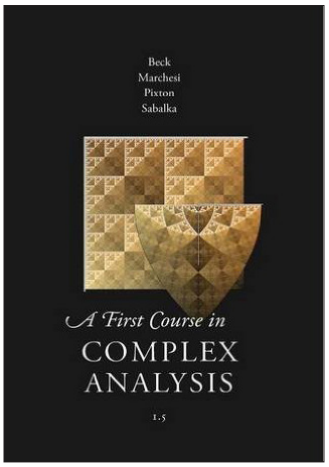
Not everyone thinks the answer is obvious, and so in 2010 we began the Open Textbook Initiative to encourage the production and adoption of open access textbooks for standard undergraduate courses in mathematics. In the nine years since then our Editorial Board has evaluated more than 100 textbooks to find those that can serve as full-fledged texts suitable for course adoption. The board looks for mathematical correctness, coverage of the usual content of the subject, a sufficient number of exercises, and evidence that the book is used by instructors other than the author alone.

Many of the books that we have listed were commercially published and then dropped by the publishers during the great consolidation of publishing houses that has taken place in the last 30 years, so that there are now just a handful of large companies in the textbook market. At the same time the advances in technology have made it possible for a single author or a small group of authors to produce excellent-looking books, distribute them over the internet, and sell printed copies through Amazon and Barnes and Noble.

There are now 53 books on our approved list. The courses represented include all of the undergraduate courses that are widely taught in U.S. and Canadian colleges and universities. Every single book on our list is free in PDF format. Many, though not all, are also available for free as online texts that can make use of interactive features. And many are also available as printed books, usually at a price far less than the commercial competitors.



Now, in 2019, a major focus of our project is working with a few of the books that have a community of users and contributors. These books show great promise to gain broad usage and to last for many years. We are encouraging authors to write their books in PreTeXt, a markup language that uses LaTeX for the mathematical expressions, and from which it is possible to produce versions in PDF, HTML, ePUB, and other formats using the same source files.

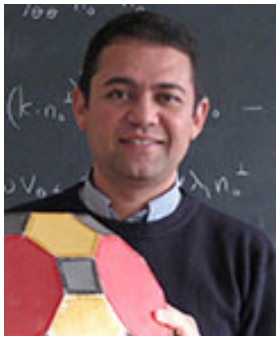


The textbook project and the development of PreTeXt are part of the UTMOST project funded by the National Science Foundation (UTMOST = Undergraduate Teaching of Mathematics with Open Software and Textbooks). ■

- Kent Morrison

AIM's New Board Members

Scientific, Human Resources, and Advisory Boards



Jesus De Loera has joined the Scientific Board. He is Professor of Mathematics at the University of California, Davis. His research areas are discrete and computational geometry with a particular interest on the combinatorial structure of convex polytopes. He received his Ph.D. from Cornell University.

Daniela Ferrero is the newest member of the Human Resources Board. She is Associate Professor of Mathematics at Texas State University in San Marcos. Her research interests are graph theory and discrete mathematics. She received her Ph.D. from the Polytechnic University of Catalonia.

Hortensia Soto is the latest addition to the Advisory Board. She is Professor of Mathematics at the University of Northern Colorado and the associate secretary of the MAA. Her research focuses on the teaching and learning of undergraduate mathematics. She received her Ph.D. from the University of Northern Colorado.

CALL FOR PROPOSALS

We are seeking proposals for week-long workshops for up to 28 people and SQuaRE collaborations for 4-6 researchers to take place in 2020-21 at AIM in San Jose, CA.

Proposals require:

- **a list of organizers**
- **a list of potential participants**
- **a description of goals**
- **an outline of how goals will be met**

Application deadline: November 1, 2019.

For more details and online applications:
www.aimath.org/research

Featured Workshop

Sarnak's Conjecture

In December of 2018 AIM hosted a workshop devoted to a conjecture of Peter Sarnak on the properties of the Möbius function. The Möbius function is one of those deceptively simple objects in mathematics that has an extremely easy-to-understand definition, yet provides a wealth of hidden information. It is denoted by μ and defined on the set of positive integers as follows. If $n = 1$, then $\mu(n) = 1$. If $n = p_1 p_2 \cdots p_k$ where the p_i are distinct primes then $\mu(n) = (-1)^k$. Othewise it is zero. For example, $\mu(2) = (-1)^1 = -1, \mu(4) = \mu(2^2) = 0, \mu(10) = \mu(2 \times 5) = (-1)^2 = 1$.

One might think that there should be more or less an equal number of positive integers with an even number of distinct prime factors as an odd number of distinct prime factors. And in an average sense that is true. If we average the values of the Möbius function starting with $n = 1$ all the way up to N, it can be proved that the average tends to zero. In symbols, as n gets large

$$\frac{1}{N}\sum_{n=1}^{N}\mu(n) \to 0.$$

It is surprising and not at all obvious that the above statement is equivalent to the prime number theorem, which tells us about how many primes there are less than N when N is large. As N gets large (or tends to infinity) the number of primes less than or equal to N is approximately $N/\log N$.

A more refined question than the above is to ask what happens to the sum $\sum_{n=1}^{N}\mu(n)$ as N gets large. The famous Riemann Hypothesis is equivalent to an assertion that the sum exhibits square root cancellation, or that for every $\epsilon > 0$,

$$\left|\sum_{n=1}^{N}\mu(n)\right| \quad < \quad C_\epsilon N^{1/2+\epsilon},$$

where C_ϵ is some constant that depends on ϵ. The exponent $1/2$ suggests that the $\mu(n)$s might exhibit random behavior, analogous to measuring the mean distance of a random walk. With the random point of view it is not unnatural to generalize these sums and ask what happens to

$$\sum_{n=1}^{N}\mu(n)\gamma(n) \quad \text{or} \quad \frac{1}{N}\sum_{n=1}^{N}\mu(n)\gamma(n)$$

where γ is some arbitrary function defined on the integers (or sequence). A known example is the case when $\gamma(n) = e^{in\alpha}$ with α a fixed real number. For this case, we also have that the average

$$\frac{1}{N}\sum_{n=1}^{N}\mu(n)e^{in\alpha} \to 0.$$

Now $e^{in\alpha}$ is a point on the unit circle obtained by a moving α radians n times from the point at $(1, 0)$. Peter Sarnak made the remarkable conjecture that under certain conditions $e^{in\alpha}$ can be replaced with $f(T^n(x_0))$ where T is a continuous self-map on a compact metric space X, f is a continuous real or complex-valued function on X, x_0 is any point in X. (In the above, $x_0 = 1$, $T(\theta) = e^{i(\theta+\alpha)}$, and f is the identity function.) In keeping with the randomness point of view, $\gamma(n)$ is seen as a sampling sequence of a dynamical system. For the expert, the actual conjecture requires that T be a map whose topological entropy is zero.

A motivation for the workshop came from recent progress on a conjecture of Chowla's. This conjecture replaces $\gamma(n)$ with shifts of the Möbius function $\mu(n+\ell_1)\mu(n+\ell_2)\cdots\mu(n+\ell_k)$, and results for these shifts are analogues of the prime k-tuple conjecture.

The workshop resulted in interesting progress on Sarnak's conjecture. The workshop organizers, Mariusz Lemańczyk and Maksym Radziwill, along with Adam Kanigowski, a participant at the workshop, confirmed the conjecture in certain cases involving the notion of rigidity in dynamics. A preprint is available at arxiv.org/abs/1905.13256.

- Estelle Basor

THE AMERICAN INSTITUTE OF MATHEMATICS

thanks

for their generous and ongoing support of our vision.

www.frys.com

THANK YOU

AIM gratefully acknowledges the following donors for their generous contributions of books, reprints, journals, and archives to the AIM library in 2018 and 2019:

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John Tate
Jean Taylor

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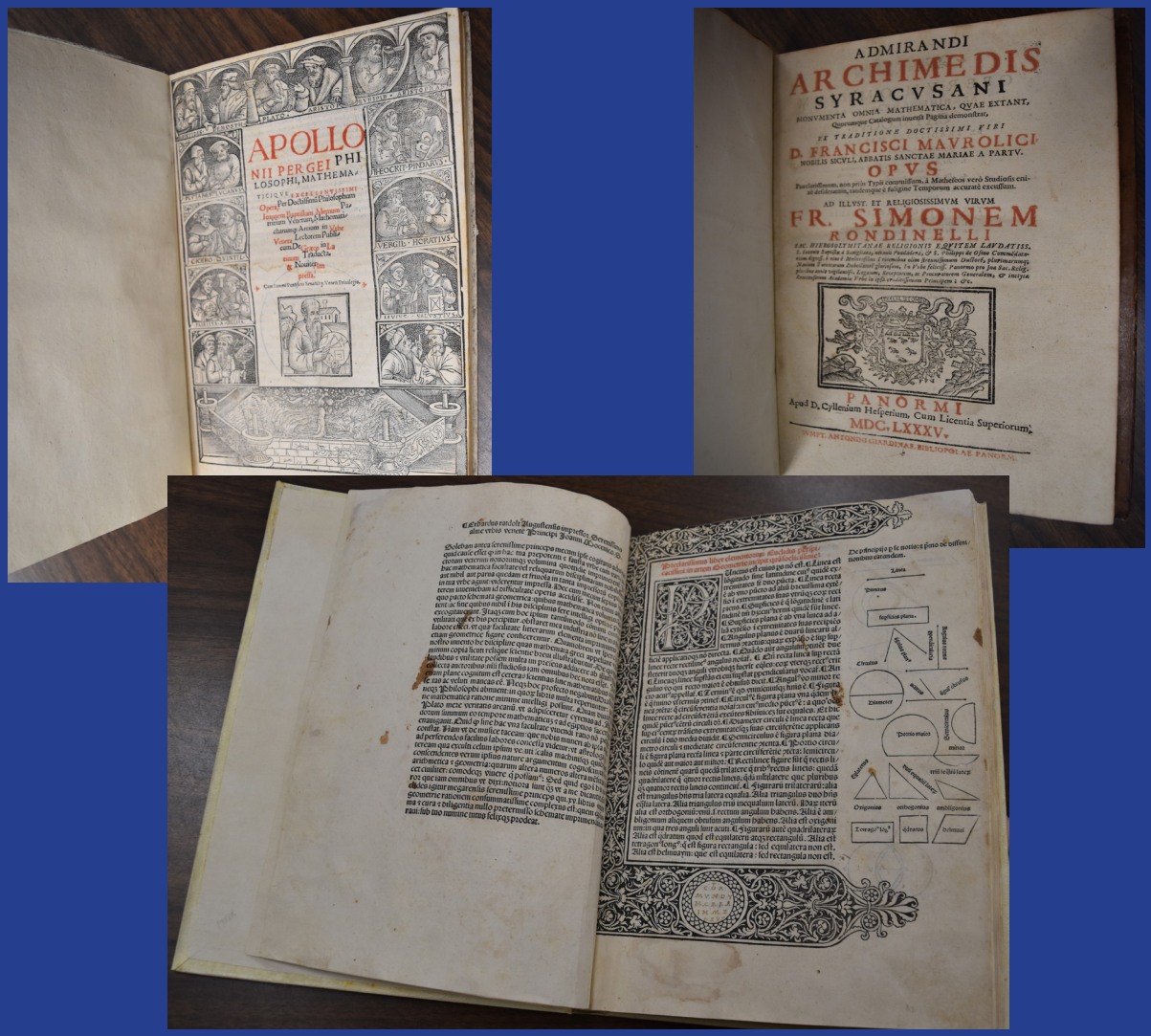
From Our Collections



Three landmarks of geometry, from top left, clockwise: the very rare editio princeps (Venice: Bernardino Bindoni, 1537) of Apollonius' Conics; the first edition (Palermo: Cyllenius Hesperius, 1685) of Francisco Maurolico's translation of Archimedes; and the editio princeps (Venice: Erhard Ratdolt, 1482) of Euclid, the first great scientific classic and the first to bear printed mathematical diagrams.